Name:

Area & Volume using π

{ 1.1 }

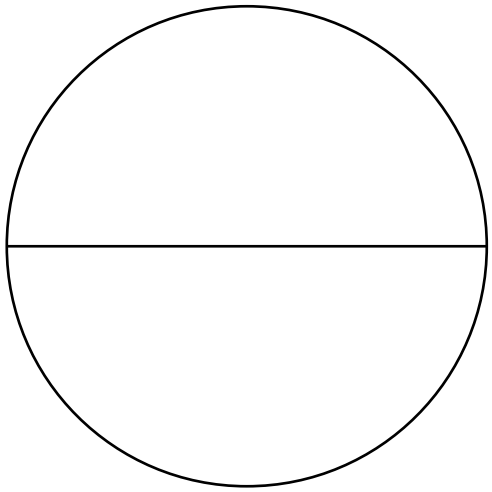
Secondary Math II Notes

OBJECTIVE: Demonstrate understanding of the irrational number π and use it to find circumference and area of a circle. Give informal arguments for the area and volume formulas of common shapes and solids.

What is π? (Formal Definition)

"Pi" is the Greek letter "p". It is represented by the symbol π. In the 1700s, mathematicians began to use this symbol to express the constant value that is defined by the ratio of the circumference of a circle to its diameter. The value of π is approximately 3.1416, but in its entirety it is an irrational number.

A Visual Approach



Diameter: The distance of the line segment that extends from one side of a circle to the other and crosses through the center point.

Radius: Half of the diameter.

Circumference: The distance around the circle. For other shapes like a rectangle or a square we call this the perimeter. This is why the Greek letter for p was originally chosen to represent the ratio.

Suppose we took a piece of string and cut it the exact length of the diameter of the circle to the left. When you take that piece of string and trace it along the circumference of the circle, how many pieces of string would you need? The answer is approximately 3.14. If you wanted to be more specific you would need to use additional digits of π.

The reason why this ratio is so impressive is because it is the ratio of the circumference to the diameter of ANY circle. It is important to remember that it is a constant, meaning it is a fixed value that never changes.

In My Own Words

Pi-

Pi is a Greek letter that is used to represent an irrational number. This number is constant and never changes. It is the number that tells me how many times the diameter of a circle can fit around the circumference of the same circle.

Circumference of a Circle

Suppose that you were given the radius of a circle and were asked to find the circumference. What steps would you take?

1. Find the diameter by multiplying the radius by 2.
2. Multiply your diameter by π to find the circumference.

$$C = 2 \cdot r \cdot \pi$$

or

$$C = 2\pi r$$

Find the circumference for each of the circles described below. Round to the nearest hundredth.

Radius: 4 cm	Radius: 7 ft	Diameter: 10 meters
$C = 2\pi r$	$C = 2\pi r$	$C = 2\pi r$
$C = 2\pi 4$	$C = 2\pi 7$	$C = 2\pi 5$
$C = 8\pi$	$C = 14\pi$	$C = 10\pi$
$C \approx 25.13\ cm$	$C \approx 43.9\ ft$	$C \approx 31.42\ m$

Rearranging Formulas

Suppose that you were given the circumference of a circle and were asked to work backwards to find the radius. What steps would you take?

1. Divide the circumference by π to find the diameter.
2. Divide the diameter by 2 to find the radius.

Notice that when you solve for r in the circumference formula on the right you are following those same steps. Divide the circumference by 2 and π to get the radius.

Solve this formula for r.

$$C = 2\pi r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

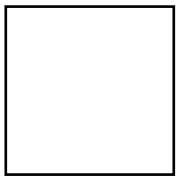
$$\frac{C}{2\pi} = r$$

Find the radius for each of the circles described below. Round to the nearest hundredth.

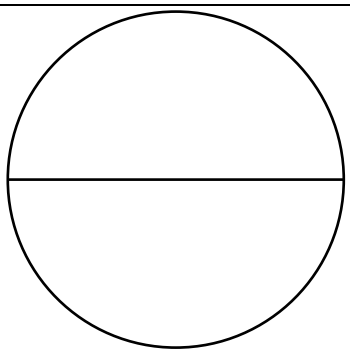
Circumference: 4 cm	Circumference: 7 ft	Diameter: 10 meters
$\frac{C}{2\pi} = r$	$\frac{C}{2\pi} = r$	$r = 5\ m$
$\frac{4}{2\pi} = r$	$\frac{7}{2\pi} = r$	
$r \approx .64\ cm$	$r \approx 1.11\ ft$	

Review of Area

Area is the measure of space within the boundaries of a 2-dimensional figure. Area is measured in square units, even if the shape of the figure is not square. For example: an area of 42 square feet can be written as 42 sq. ft. or 42 ft^2.

Shape:	Area Formula:	Example:
	$A_{square} = s^2$	Find the area of a square with a side of 4 cm. $A = 16\ cm^2$
	$A_{rec} = l \cdot w$	Find the area of a rectangle with a length of 2 feet and a width of 1.5 feet. $A = 3\ ft^2$
	$A_{para} = b \cdot h$	Find the area of a parallelogram with a base of 4 m and a height of 1.2 m. $A = 4.8\ m^2$

Challenge



Radius: 2 units

Instructions:

Consider the circle on the left. Use what you already know to make an estimate for area of this circle. Explain your reasoning by using words, sketches, or both.

Watch the video at https://learnzillion.com/lessons/2356-informally-prove-the-area-of-a-circle to get an idea of several ways to estimate the area of circle and eventually, to give and informal proof of the area formula.

Area of a Circle

Formula:	Example:
$A = \pi r^2$	Find the area of a circle that has a diameter of 5 inches. Remember that the formula for area calls for the radius not the diameter so you will need to recognize first that the radius is 2.5. Then, plug it into the formula to get an approximation of $19.63\ in^2$.

Review of Volume

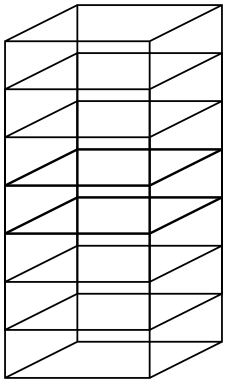
Volume is the measure of space that is enclosed within the boundaries of 3-dimensional solid. Volume is measured in cubic units, even if the shape of the figure is not a cube. For example: a volume of 42 cubic feet can be written as 42 cu. ft. or $42\ ft^3$.

Cavalieri's Principle

Bonaventura Cavalieri

The total volume of a three dimensional solid is the sum of all of the slices across that three dimensional solid.

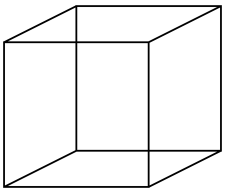
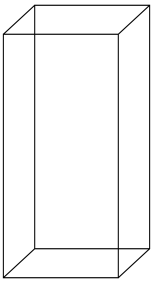
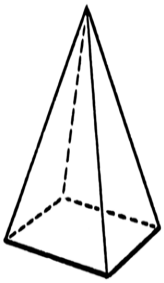
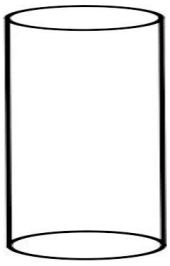
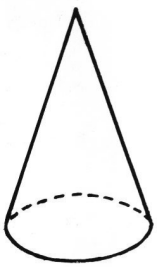
In My Own Words



If the shape of the solid is the same all the way up from bottom to top, then I can find the volume by finding the area of one slice and multiplying it by the height of the solid. For example, in a rectangular prism I can find the area of the base by multiplying length and width, then I continue on to multiply by height.

Use Cavalieri's Principle to make a conjecture for the volume formula of a cylinder. It is simply the area of the base which we know to be $\pi * r^2$, then we multiply by the height.

Watch the video at https://learnzillion.com/lessons/2451-relate-the-volume-of-prisms-cylinders-to-pyramids-cones to see how we can make a guess at the formulas for a pyramid and a cone.

Solid:	Volume Formula:	Example:
	$V_{cube} = s^3$	Find the volume of a cube with a side of 4 cm. $V = 64\ cm^3$
	$V_{rec.prism} = l \cdot w \cdot h$	Find the volume of a rectangular prism with a length of 2 feet, a width of 1.5 feet, and height of 4 feet. $V = 12\ ft^3$
	$V_{pyr} = \frac{1}{3} \cdot l \cdot w \cdot h$	Find the volume of a pyramid whose base has a length of 2 feet and a width of 1.5 feet and whose height is 4 feet. $V = 4 ft^3$
	$V_{cyl} = \pi \cdot r^2 \cdot h$	Find the volume of a cylinder whose height is 5 centimeters and whose base has a radius of 2 centimeters. $V \approx 62.83\ cm^3$
	$V_{cone} = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$	Find the volume of a pyramid whose height is 5 centimeters and whose base has a radius of 2 centimeters. $V \approx 20.94\ cm^3$