Name:

# Review of Radicals {1.2}

Secondary Math II Notes

**OBJECTIVE:** Write radicals in exact simplest form and make an approximation of the value mentally and using technology. Classify radicals as rational or irrational. Combine radical expressions using addition, subtraction, and multiplication.

## What is a Radical

$$\sqrt[2]{16} = 4$$

- Index
- Radicand Symbol
- Radicand
- Value

## Challenge

$\sqrt[2]{16} = 4$

$\sqrt[3]{8} = 2$

$\sqrt[4]{81} = 3$

$\sqrt[2]{25} = 5$

$\sqrt[6]{1} = 1$

$\sqrt[3]{64} = 4$

$\sqrt[2]{0} = 0$

Instructions:

The list to the left includes seven examples of radical expressions and their values. In your own words, describe the relationship between the index, the radicand, and the value in the given examples. Be sure that that the description is true for every case.

There are two ways to describe the relationship that we see here. Perhaps the easiest thing to say is that if we take the final value and raise it to the power of the index then we will receive the radicand. The second option is a bit more complex. We will start with the radicand and work our way to the value. First we factor the radicand completely. Then we use the index as an exchange rate (like currency) that allows us to remove factors from the radicand. For example, if the index is 3 then the exchange rate is 3 to 1. We use the index in this way to exchange as many factors as possible, removing them from the radicand a group at a time. If the index were 3 then we would exchange a group of size three for a group of size one as it is removed. Then we multiply all of the removed factors to get the final value. For example, consider a radicand with a radical 25 and an index of 2. We factor the 25 into a set of 2 fives. Since the exchange rate is 2 to 1 in this case, we exchange that set of 2 fives, for a single five, which is our final value.

## RADICALS TO KNOW

| | |
|---|---|
| $\sqrt[2]{0} = 0$ | $\sqrt[3]{0} = 0$ |
| $\sqrt[2]{1} = 1$ | $\sqrt[3]{1} = 1$ |
| $\sqrt[2]{4} = 2$ | $\sqrt[3]{8} = 2$ |
| $\sqrt[2]{9} = 3$ | $\sqrt[3]{27} = 3$ |
| $\sqrt[2]{16} = 4$ | $\sqrt[3]{64} = 4$ |
| $\sqrt[2]{25} = 5$ | $\sqrt[3]{125} = 5$ |
| $\sqrt[2]{36} = 6$ | $\sqrt[3]{216} = 6$ |
| $\sqrt[2]{49} = 7$ | $\sqrt[3]{1000} = 10$ |
| $\sqrt[2]{64} = 8$ | |
| $\sqrt[2]{81} = 9$ | |

## Simplifying Radicals

1. -Factor the radicand.
2. -Use the exchange rate to remove as many factors as possible from the radicand. Leave any factors that cannot be exchanged.
3. -Multiply the factors back together, keeping separate those that have been removed.

| | | |
|---|---|---|
| $\sqrt[3]{16}$ | $\sqrt[2]{360}$ | $\sqrt[4]{625}$ |
| $= 2\sqrt[3]{2}$ | $= 6\sqrt[2]{10}$ | $= 5$ |

## Rational and Irrational Radical Expressions

Rational- Radicals are rational when all of the factors can be removed from the radicand using the index as the exchange rate. The square root of perfect squares (like the square root of 16) and the cube root of perfect cubes (like the cube root of 8) are good examples of this.

Irrational- Radicals are irrational when some factors remain in the radicand after simplification. An example of this is the square root of 18 (a set of threes can be exchanged, but a 2 remains). Another name for an irrational radical of this type is "surd". These radicals have an exact form (same as simplified form), but they can also be approximated on your calculator. Just remember that the digits that appear on your calculator will go on forever and when you round you change from an exact answer to an estimated answer.

### Practice

| Radical Expression | Exact Simplified Value | Estimated Value | Rational or Irrational |
|---|---|---|---|
| $\sqrt[2]{100}$ | $=10$ | $NA$ | R |
| $\sqrt[3]{54}$ | $=3\sqrt[3]{2}$ | $\approx 3.78$ | I |
| $3\sqrt[2]{200}$ | $=30\sqrt[2]{2}$ | $\approx 42.43$ | I |
| $5\sqrt[2]{49}$ | $=35$ | $NA$ | R |
| $\sqrt[3]{32}$ | $=2\sqrt[3]{4}$ | $\approx 3.17$ | I |
| $\sqrt[2]{1000}$ | $=10\sqrt[2]{10}$ | $\approx 31.62$ | I |

## Adding & Subtracting Radicals

**Like Terms:** In algebra like terms are terms that contain the same variables raised to the same powers. Like terms can be combined into a single term using addition and subtraction. In this case the coefficients reflect the change but the variable and its power stay the same.

Simplify the following expressions by adding like terms:

$2x+3x=$

$5x^2-2x^2=$

$2x+4y=$

$2x+3x^2=$

$4y+y=$

$2x+3x=5x$, notice that the power of the variable does not change.

$5x^2-2x^2=3x^2$, notice again that the power of the variable does not change.

$2x+4y=2x+4y$, these are not like terms because the variables are not the same.

$2x+3x^2=2x+3x^2$, these are not like terms because the power of the variable is not the same.

$4y+y=5y$, remember that a single y has a coefficient of 1.

**Like expressions:** When dealing with radicals, like radical expressions are expressions that have the same radicands and indices (plural for index). These expressions can be combined into a single expression when using addition or subtraction. In such a case the numbers outside the radical reflect the change but the radicand and index stay the same.

Identify and combine the like terms in the following expression:

$$4\sqrt[3]{3}+2+\sqrt[3]{8}-4\sqrt[2]{3}+\sqrt[3]{4}-1+2\sqrt[2]{11}-\sqrt[3]{3}-21\sqrt[2]{3}+4\sqrt[2]{11}$$

$$=3\sqrt[3]{3}+1+\sqrt[3]{8}-25\sqrt[2]{3}+\sqrt[3]{4}+6\sqrt[2]{11}$$

| | | |
|---|---|---|
| $4\sqrt[3]{5}+2\sqrt[3]{5}-\sqrt{5}=$ <br> $=6\sqrt[3]{5}-\sqrt{5}$ | $7\sqrt{8}+\sqrt{2}+2\sqrt{18}=$ <br> $=7\sqrt{8}+\sqrt{2}+6\sqrt{2}$ <br> $=7\sqrt{8}+7\sqrt{2}$ | $\sqrt{147}+2\sqrt{192}-4\sqrt{3}+\sqrt{75}=$ <br> $=7\sqrt[3]{3}+16\sqrt[3]{3}-4\sqrt[3]{3}+5\sqrt[3]{3}$ <br> $=23\sqrt[3]{3}$ |

## Rules for Adding & Subtracting Radicals

- Simplify all radicals first.
- Remember that like expressions must have the same radicands and indices.
- The radicand and index should NOT change in the addition or subtraction process.

| Multiplication Property for Radicals |
|---|
| The product of two radical expressions of the form $a\sqrt[b]{c}$ & $d\sqrt[b]{e}$ is $a \cdot d\sqrt[b]{c \cdot e}$. |
| **In My Own Words** |

To multiply two radical expressions they must share the same index. We will multiply the numbers outside of the radical symbol together and multiply the numbers inside the radicand together.

| Practice | | | |
|---|---|---|---|
| Radical Expression | Exact Simplified Value | Estimated Value | Rational or Irrational |
| $3\sqrt[3]{2} \cdot 5\sqrt[3]{4} =$ | $= 15\sqrt[3]{8}$<br>$= 15 \cdot 2$<br>$= 30$ | $NA$ | R |
| $-2\sqrt{22} \cdot 6\sqrt{4} =$ | $= -12\sqrt{88}$<br>$= -24\sqrt{22}$ | $\approx 112.57$ | I |
| $2\sqrt{6} \cdot 3\sqrt{12} =$ | $= 6\sqrt{72}$<br>$= 6 \cdot 6\sqrt{2}$<br>$= 36\sqrt{2}$ | $\approx 50.91$ | I |