Name:

Integer Exponent Properties {1.3}

Secondary Math II Notes

OBJECTIVE: Use patterns to identify integer exponent properties. Represent properties in symbols and words and use them to simplify expressions.

Equivalent Forms

Use the definition of an exponent to write expressions that are equivalent (equal) to the ones below.

$x^7 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$	$\alpha \cdot \alpha \cdot \alpha \cdot \alpha = \alpha^4$

All integer exponent properties are an expansion of this definition. We will use this definition as a building block to acquire knowledge about other properties.

Multiplication Property

Rewrite each expression using only your knowledge of the definition of an exponent.

$y^3 \cdot y^2 = \quad y \cdot y \cdot y \cdot y \cdot y = y^5$	$a^3 \cdot a = \quad a \cdot a \cdot a \cdot a = a^4$
$m \cdot m^3 \cdot m^5 = m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m = m^9$	$b^2 \cdot b^6 = \quad b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = b^8$

Property:

$$x^a \cdot x^b = x^{(a+b)}$$

In words:

When we multiply two terms with the same base we add the exponents.

$w^3 \cdot w^{34} = \quad w^{37}$	$a^3 \cdot a \cdot a^{12} = \quad a^{16}$	$x^m \cdot x^n = \quad x^{m+n}$

Power to a Power Property

Rewrite each expression using only your knowledge of the definition of an exponent and the multiplication property.

$(m^3)^2 = m^3 \cdot m^3 = m^6$	$((x^2)^2)^3 = ((x^2)^2)^3 = x^2x^2x^2x^2x^2x^2 = x^{12}$
$(\theta^2)^4 = \theta^2 \cdot \theta^2 \cdot \theta^2 \cdot \theta^2 = \theta^8$	$(y^5)^1 = y^5$

Property:

$$\left(x^a\right)^b = x^{(a \cdot b)}$$

In words:

When a power is applied to a base with a preexisting power, the result is the base raised to the product of the two powers.

$(w^4)^{12} = \quad w^{48}$	$((x^2)^4)^3 = \quad x^{24}$	$((w^a)^b)^c = \quad w^{abc}$

Distributive Power Property

Rewrite each expression using only your knowledge of the definition of an exponent and the multiplication property.

$(2y^3)^2 = \quad 2y^3 \cdot 2y^3 = 4y^6$	$(5x^3w^3)^2 = \quad 5x^3w^3 \cdot 5x^3w^3 = 25x^6w^6$
$(4z^2)^3 = \quad 4z^2 \cdot 4z^2 \cdot 4z^2 = 64z^6$	$(4m^{10})^2 = \quad 4m^{10} \cdot 4m^{10} = 16m^{20}$

Property:

$$\left(mx^a\right)^b = m^b \cdot x^{(a \cdot b)}$$

In words:

When multiple items within a set of parentheses are raised to a power, each item is individually raised to that power.

$(6w^4)^2 = \quad 36w^8$	$(3xy)^3 = \quad 27x^3y^3$	$(5w^{11})^2 = 25w^{22}$

Division Property of Exponents

Rewrite each numerator and denominator using the definition of an exponent. Then simplify by finding quotients equal to 1.

$\frac{z^5}{z^2} = \quad \frac{zzzzz}{zz} = z^3$	$\frac{x^5}{x} = \quad \frac{zzzzz}{zz} = z^3$
$\frac{w^3}{w^2} = \quad \frac{www}{ww} = w^1$	$\frac{a^5}{a} = \quad \frac{aaaaa}{a} = a^4$

Property:

$$\frac{x^a}{x^b} = x^{a-b}$$

In words:

When we divide two terms with the same base we subtract the bottom power from the top power.

$\frac{(ab)^5}{(ab)^3} = \quad (ab)^2$	$\frac{6x^{105}}{3x^{10}} = \quad 2x^{95}$	$\frac{x^3y^{11}}{xy^6} = \quad x^2y^5$

Negative Power Property

We will use two different methods to simplify the same expression. The results will give us an idea about negative exponents.

Simplify using the division property.	Rewrite each numerator and denominator using the definition of an exponent. Then simplify by finding quotients equal to 1.
$\frac{y^3}{y^7} = \quad y^{-4}$	$\frac{y^3}{y^7} = \quad \frac{yyy}{yyyyyyy} = \frac{1}{y^4}$
$\frac{x}{x^7} = \quad x^{-6}$	$\frac{x}{x^7} = \quad \frac{x}{xxxxxxx} = \frac{1}{x^6}$

What About $\frac{1}{x^{-3}}$?

well we know that $x^{-3} = \frac{1}{x^3}$

so, $\frac{1}{x^{-3}} = \frac{1}{\frac{1}{x^3}} = \frac{\frac{1}{1}}{\frac{1}{x^3}} = \frac{1}{1} \cdot \frac{x^3}{1} = x^3$

Property:

$$x^{-a} = \frac{1}{x^a} \quad and \quad \frac{1}{x^{-a}} = x^a$$

In words:

When a base has a negative power in the numerator it is equivalent to the expression where the base has a positive power in the denominator.

When a base has a negative power in the denominator it is equivalent to the expression where the base has a positive power in the numerator.

"Jump the fraction line to change the exponent's sign."

$\frac{-2x^{-5}}{4} = \quad \frac{-1}{2x^5}$	$\frac{x^4y^5}{x^6y^{-2}} = \quad \frac{y^5y^2}{x^2} = \frac{y^7}{x^2}$	$\frac{x^{-3}y^{-11}}{x^{-1}y^{-6}} = \quad \frac{x^1y^6}{x^3y^{11}} = \frac{1}{x^2y^5}$

Practice & Review

$\frac{x^4x^5y^{-2}}{x^6y} = \quad \frac{x^9}{x^6y^1y^2} = \frac{x^3}{y^3}$	$\frac{14x^4y^3}{21x^{12}y^4z^5} = \quad = \frac{2}{3x^8y^1z^5}$

Zero Power Property

Property:

$$for\ any\ x \neq 0, \qquad x^0 = 1$$

In words:

Any non-zero base raised to the power of 0 is equivalent to 1.

Justification #1 - Using Multiplication

$x^3 * x^0 = \quad x^{3+0} = x^3$	Suppose that we are unsure what the value of x^0 is. All we know is that a term multiplied by this value gives us the same term back. The value must be 1.

Justification #2 - Using Division

$\frac{w^5}{w^0} \quad w^{5-0} = w^5$	Suppose that we are unsure what the value of x^0 is. All we know is that a term divided by this value gives us the same term back. The value must be 1.

Justification #3 – A Comparison

Simplify using the division property.	Simplify by expanding and finding quotients equal to 1.	We know that the expressions are equal, even if we simplify them in different ways. So $y^0=1$.
$\frac{y^5}{y^5} = y^{5-5} = y^0$	$\frac{y^5}{y^5} = \frac{yyyyy}{yyyyy} = 1$	

Justification #4 – Observing a Pattern

$3^3 = 27$ $3^2 = 9$ $3^1 = 3$ $3^0 = 1$ $3^{-1} = \frac{1}{3}$	We can see that the pattern in the values on the right shows division by 3 as the power on the left decreases by one. So we would expect that after three the next value should be 1.

Simplifying Expressions

1	Distribute powers to all bases within parentheses if necessary.
2	Use the multiplication property to combine like bases that are either both in the numerator or denominator.
3	Change any negative powers to positive powers by using the negative exponenet property.
4	Use the division property to insure that only one of every base is left in the expression.

$\left(\frac{y^{-1}z^5}{y^5}\right)^{-1} = \left(\frac{y^1z^{-5}}{y^{-5}}\right) = \frac{y \cdot y^5}{z^5} = \frac{y^6}{z^5}$	$\frac{(3x^2)^3a^{-12}b^2}{(a^{-2})^{-1}b^3x^0} = \frac{27x^6a^{-12}b^2}{a^2b^3} = \frac{27xb^2}{a^2a^{12}b^3} = \frac{27x}{a^{14}b}$