Name:

Rational Exponents {1.4}

Secondary Math II Notes

OBJECTIVE: Explain the meaning of a rational exponent by extending the properties of integer exponents. Rewrite expressions involving radicals and rational exponents. Simplify expressions involving rational exponents.

What are Rational Exponents?

4^0	1	Use the information in this table to make an estimate for the value of $4^{\frac{3}{2}}$. Explain your reasoning.	Since 3/2 is between 1 and 2. I estimate that the value should be between 4 and 16. I could make an estimate of 10.
4^1	4		
4^2	16		
4^3	64		

Exploration

Use the exponent properties that you know to simplify the expressions below. Record your observations.

$49^{\frac{1}{2}} \cdot 49^{\frac{1}{2}} =$	$27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} =$	$16^{\frac{1}{4}} \cdot 16^{\frac{1}{4}} \cdot 16^{\frac{1}{4}} \cdot 16^{\frac{1}{4}} =$	$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} =$
$= 49^1 = 49$	$= 27$	$= 16$	$= 5$

Observations

It seems that 49 to the power of ½ is equal to 7, since 7 is the only number that can multiply by itself to get 49. From my previous knowledge, I know that the relationship between 7 and 49 is that 7 to the power of 2 equals 49, or that the square root of 49 is 7.

As I continue my exploration it seems that the power of ½ is associated with a square root and the power of 1/3 is associated with a cube root and so on....

Property of Rational Exponents (1)

$$x^{\frac{1}{b}} = \sqrt[b]{x}$$

In My Own Words

When a base has an exponent that has 1 for the numerator and another number for the denominator, that expression is equivalent to a radical expression where the base is used for the radicand and the denominator is used for the index.

Exponential form to Radical form		
$(17x)^{\frac{1}{2}} = \sqrt[2]{17x}$	$(2^3)^{\frac{1}{4}} = \sqrt[4]{2^3}$	$(xy)^{\frac{1}{3}} = \sqrt[3]{xy}$
$5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{5}}$	$22^{\frac{2}{10}} = 22^{\frac{1}{5}} = \sqrt[5]{22}$	$(10)^{\left(\frac{1}{10}\right)} = \sqrt[10]{10}$

Radical form to Exponential form		
$\sqrt{17x} = (17x)^{\frac{1}{2}}$	$\sqrt[3]{x} = x^{\frac{1}{3}}$	$\sqrt{xy}^{3} = \left((xy)^{\frac{1}{2}}\right)^{3} = (xy)^{\frac{3}{2}}$
$\sqrt{13^3} = (13^3)^{\frac{1}{2}} = 13^{\frac{3}{2}}$	$\sqrt[5]{21z} = (21z)^{\frac{1}{5}}$	$21\sqrt[5]{z} = 21z^{\frac{1}{5}}$

Challenge
$$\sqrt[5]{\sqrt{3}} = \sqrt[10]{3}$$ The statement above is true. Give mathematical evidence to support it using properties that you know.
$\sqrt[5]{\sqrt{3}}$ $= \left((3)^{\frac{1}{2}}\right)^{\frac{1}{5}}$ $= 3^{\frac{1}{10}}$ $= \sqrt[10]{3}$

Challenge

$9^{\frac{3}{2}}$	27
$4^{\frac{5}{2}}$	32
$1^{\frac{8}{3}}$	1
$8^{\frac{2}{3}}$	4
$16^{\frac{3}{4}}$	8

We know that $25^{\frac{1}{2}} = 5$ but we have not yet learned what happens when the numerator of the exponent is a number other than one. Make an inference about the role of the number in the numerator by observing this table.

I know that the denominator should be used as the index in a radical expression. When I take the square root of 9 I get 3. If I raise that 3 to the power of the numerator I get 27! We can try this pattern again to make sure that it works. If I take the square root of 4 I get 2. If I raise 2 to the power of the numerator I get 32.

Property of Rational Exponents (2)

$$x^{\frac{a}{b}} = \left(x^{\frac{1}{b}}\right)^{a} = \left(x^{a}\right)^{\frac{1}{b}} = \sqrt[b]{x^{a}} = \sqrt[b]{x}^{\,a}$$

In My Own Words

When a base has an exponent that is rational it can be written as a radical where the base is used as the radicand, the numerator is the exponent, and the denominator is the index.

Exponential form to Radical form

$(5x)^{\frac{3}{2}} = \sqrt[2]{5x}^{3}$	$(2^3)^{\frac{3}{4}} = \sqrt[4]{\left(2^3\right)^3} = \sqrt[4]{2^9}$	$(xy^4)^{\frac{1}{3}} = \sqrt[3]{xy^4}$
$5^{-\frac{5}{2}} = \frac{1}{5^{\frac{5}{2}}} = \frac{1}{\sqrt[2]{5^5}}$	$22^{\frac{2}{10}} = 22^{\frac{1}{3}} = \sqrt[3]{22}$	$(10)^{\left(\frac{10}{10}\right)} = 10^1 = 10$

Radical form to Exponential form

$\sqrt{17x^5} = \left(17x^5\right)^{\frac{1}{2}}$	$\sqrt[3]{x^3} = x^{\frac{3}{3}} = x^1 = x$	$\sqrt{xy}^{3} = \left(xy\right)^{\frac{3}{2}}$
$\sqrt{13^3} = 13^{\frac{3}{2}}$	$\sqrt[5]{\left(21z\right)^4} = \left(21z\right)^{\frac{4}{5}}$	$21\sqrt[5]{z^4} = 21z^{\frac{4}{5}}$

Changing Form for Organization

Organize the following expression in order from least to greatest.

$$9^{\frac{1}{2}},\ 9^{\frac{3}{2}},\ \sqrt{3},\ \sqrt[3]{9}$$

$$\left(3^2\right)^{\frac{1}{2}},\ \left(3^2\right)^{\frac{3}{2}},\ 3^{\frac{1}{2}},\ \left(3^2\right)^{\frac{1}{3}}$$

$$3^1,\quad 3^3,\quad 3^{\frac{1}{2}},\quad 3^{\frac{2}{3}}$$

$$3^{\frac{1}{2}},\quad 3^{\frac{2}{3}},\quad 3^1,\quad 3^3$$

Challenge- Changing Form for Simplification

Recall that we may only multiply and divide radicals with like radicands and exponential expressions with like bases. Keep this in mind as you simplify the following expressions as much as possible.

$$\sqrt[3]{25}\cdot\sqrt{5^3}$$

$$=\left(5^2\right)^{\frac{1}{3}}\cdot 5^{\frac{3}{2}}$$

$$=5^{\frac{2}{3}}\cdot 5^{\frac{3}{2}}$$

$$=5^1$$

$$\frac{\sqrt[4]{3}}{\sqrt{3}}$$

$$=\frac{3^{\frac{1}{4}}}{3^{\frac{1}{2}}}$$

$$=3^{-\frac{1}{2}}$$

$$=\frac{1}{3^{\frac{1}{2}}}$$

$$\sqrt[4]{5^2}$$

$$=5^{\frac{2}{4}}$$

$$=5^{\frac{1}{2}}$$

$$\sqrt[4]{\sqrt[5]{\sqrt[2]{3}}}$$

$$=\left(\left(3^{\frac{1}{2}}\right)^{\frac{1}{5}}\right)^{\frac{1}{4}}$$

$$=3^{\frac{1}{40}}$$