Name:

The Imaginary Unit (Non-Real Numbers)

Secondary Math II Notes

{ 1.6 }

OBJECTIVE: Understand the need for the imaginary unit. Simplify expressions using the definition of the imaginary unit. Read and write numbers in complex form.

The Need for an Imaginary Unit

If $x^2 = 9$, then what is x?	If $x^2 = 15$, then what is x?	If $x^2 = -1$, then what is x?
x=3 or x=-3	x=the positive or negative square root of 15. Both will multiply by themselves to give 15.	There does not exist a real number that will satisfy this equation. However, there is a number in the complex number system beyond the real number system that you are familiar with (called i) that can satisfy this equation. In fact, that is its exact definition. $i^2=-1$

Definition of the Imaginary Unit

$i^2 = -1$	$\sqrt{-1} = i$

In My Own Words

The imaginary unit "i" is a number beyond the real number system. We only know one thing about it, when we square it we get -1. It cannot be graphed on a real number line.

Simplifying Expressions

i^4 $= 1$	$\sqrt{-25}$ $= 5i$	$\sqrt{-9}$ $= 3i$	$-\sqrt{-16}$ $= -4i$
$\sqrt{-32}$ $= 4i\sqrt{2}$	$(2i)^3$ $= -8i$	$\sqrt{150-250}$ $= 10i$	$3i^2$ $= -3$
$\sqrt{4^2-4(2)(8)}$ $= 4i\sqrt{3}$	$-\sqrt{24}$ $= -2\sqrt{6}$	$(-i)^2$ $= -1$	$-i^8$ $i \cdot i \cdot i \cdot i$

Making Use of Patterns

Complete the table below using the fact that $i^2 = -1$.

i	i	i	Observations:
i^2	$i \cdot i$	-1	As the exponents go up by one, the resulting simplification seems to follow a pattern. i, -1, -i, 1, ...and so on.
i^3	$i \cdot i \cdot i$	$-i$	
i^4	$i \cdot i \cdot i \cdot i$	1	
i^5	$i \cdot i \cdot i \cdot i \cdot i$	i	
i^6	$i \cdot i \cdot i \cdot i \cdot i$	-1	

Develop a strategy to simplify i^{103} by using your observations of the table above. Give the value of i^{103} and explain your reasoning.

i^{103} is equal to $-i$. This is because every group of four simplifies to 1 and multiplying by one, no matter how many times, will not have an impact on the final product. So we can take the total number of imaginary units, in this case 103 and divide by 4. There are 25 groups of 4 with 3 units left over. The groups of four will multiply to one, and the remaining units are easy to calculate. $i^3 = -i$

Challenge

Consider the following equation.

$$x = \sqrt{p - q}$$

Choose a number for p and a number for q. Use these values and the equation above to determine whether x will be real or imaginary. Be sure to show your work. Repeat this process 3 more times. What is the rule for the relationship between p and q when the result is real? when the result is imaginary? Consider the case when p and q are negative numbers. Does the rule still hold?

p	q	Work	Result (is x real or imaginary)
8	3	$x = \sqrt{8-3}$ $x = \sqrt{5}$	Real

Results:

You will obviously have different results depending on the numbers that you choose. Regardless, it should be clear that when p is greater that (or equal) q the result is real, but if p is less that q then the radicand is negative and the result is imaginary. The rule still holds when the numbers are negative.