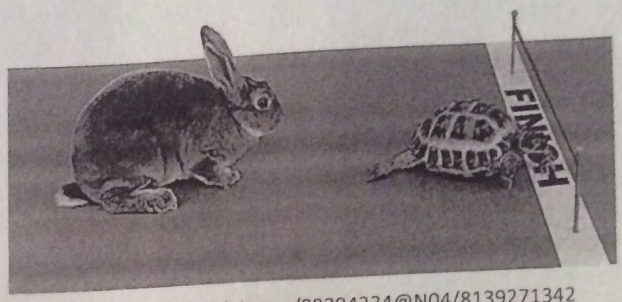
1.6 The Tortoise and the Hare

A Solidify Understanding Task



In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$d = t^2$ (d in meters and t in seconds) Quadratic function

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d = 2^t$ (d in meters and t in seconds) Exponential Function

1. At what time does the hare catch up to the tortoise?

t	Hare	Tortoise
0	0	1
1	1	2
2	4	4
3	9	8

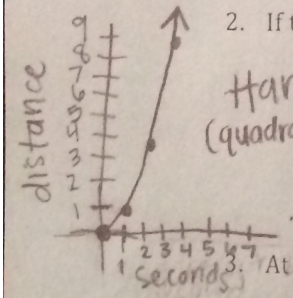
$t=0$ $d=(0)^2=0$ $d=2^0=1$

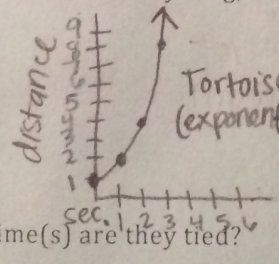
$t=1$ $d=(1)^2=1$ $d=2^1=2$

$t=2$ $d=(2)^2=4$ $d=2^2=4$

At $t=2$ secs, the Hare catches the tortoise.

2. If the race course is very long, who wins: the tortoise or the hare? Why?





The tortoise wins. I know that because when I look at the graph as t increases, the function for the tortoise increases faster.

3. At what time(s) are they tied?

They are tied at $t=2$ sec. and $t=4$ sec.

time	H	T
0	0	1
1	1	2
2	4	4
3	9	8
4	16	16
5	25	32
6	36	64

4. If the race course were 15 meters long who wins, the tortoise or the hare? Why?

$d=15$

Hare
$15=t^2$
$\sqrt{15}=\sqrt{t^2}$
$3.87=t$
The Hare reaches 15m at 3.87 sec.

Tortoise
$t=3.87$
$d=2^{3.87}$
$d=14.$
The tortoise hasn't reached 15 m at 3.87 sec.

The Hare.
The tortoise won't catch the hare until they reach 16 meters.

5. Use the properties $d = 2^t$ and $d = t^2$ to explain the **speeds** of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$
[0, 2)	$t=0 \rightarrow 1$ m $t=2 \rightarrow 4$ m He went 3m in 2 sec $3m/2sec \rightarrow 1.5m/1sec$	$t=0 \rightarrow 0$ m $t=2 \rightarrow 4$ m He went 4m in 2 sec $4m/2sec \rightarrow 2m/sec$
[2, 4)	$t=2 \rightarrow 4$ m $t=4 \rightarrow 16$ m He went 12m in 2 sec. $12m/2sec. \rightarrow 6m/sec.$	$t=2 \rightarrow 4$m $t=4 \rightarrow 16$m He went 12 m in 2 sec. $12m/2sec. \rightarrow 6m/sec.$
[4, ∞) ↑ use big # like 26	$t=4 \rightarrow 16$ m $t=20 \rightarrow 400$ m He went 384 m in 16 sec.	$t=4 \rightarrow 16$m $t=20 \rightarrow 1,048,576$m He went 1,048,560m in 16 sec.

* These are <u>average</u> speeds over the time intervals, these speeds aren't exact speeds for a specific time.