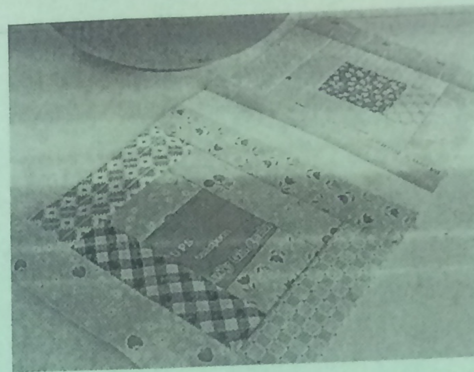
2.4 A SQUARE DEAL

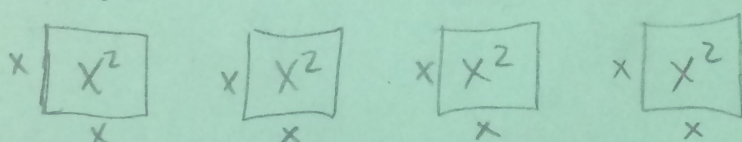
A Solidify Understanding Task



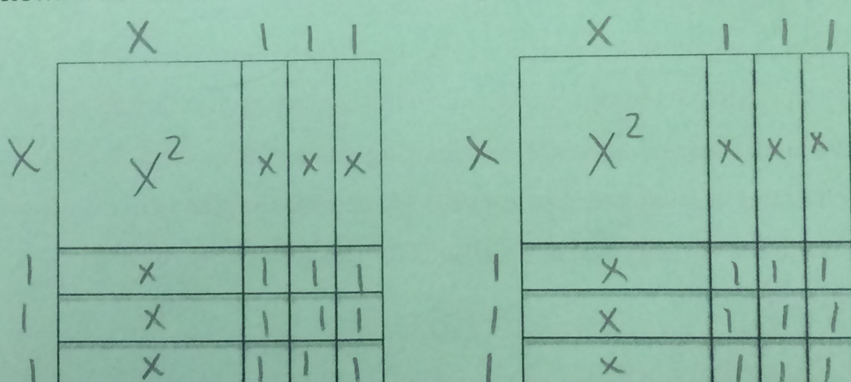
Quadratic Quilts, Revisited

Remember Optima's quilt shop? She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$. Model this area expression with a diagram.



2. One of the customer service representatives finds an envelope that contains the blocks pictured below. Write the order that shows two equivalent equations for the area of the blocks.



$A(x) = 2\underbrace{(x+3)}_{\text{side length of each square}}{}^2$ — 2 squares ordered

$A(x) = 2\underbrace{(x^2+6x+9)}_{\text{pieces inside one square}}$ — 2 squares ordered

$A(x) = \underbrace{2x^2+12x+18}_{\text{pieces inside both squares}}$

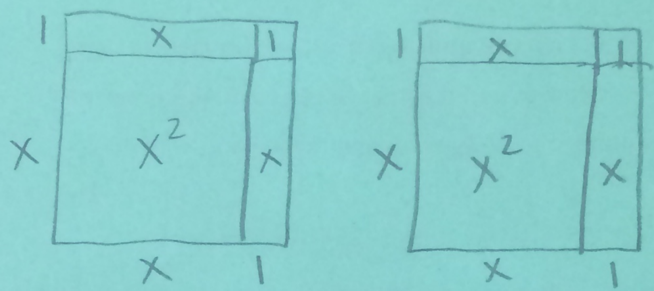
basic block: $\boxed{x^2}$

3. What equations for the area could customer service write if they received an order for 2 blocks that are squares and have both dimensions increased by 1 inch in comparison to the basic block? Write the area equations in two equivalent forms. Verify your algebra using a diagram.

$A(x) = 2(x+1)^2$

$A(x) = 2(x^2+2x+1)$

$2x^2+4x+2$

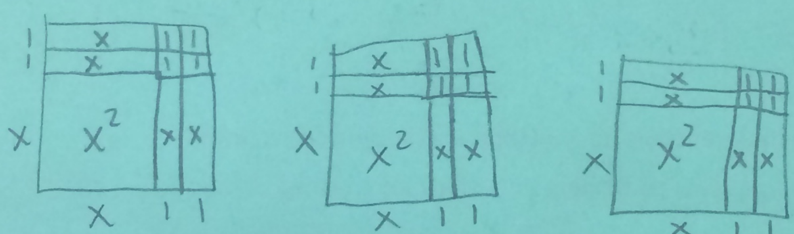


4. If customer service receives an order for 3 blocks that are each squares with both dimensions increased by 2 inches in comparison to the basic block? Again, show 2 different equations for the area and verify your work with a model.

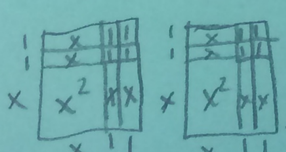
$A(x) = 3(x+2)^2$

$A(x) = 3(x^2+4x+4)$

$3x^2+12x+12$



5. Clementine is at it again! When is that dog going to learn not to chew up the orders? (She also chews Optima's shoes, but that's a story for another day.) Here are some of the orders that have been chewed up so they are missing the last term. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.



$2x^2 + 8x + 8$

$2(x^2+4x+4)$

$2(x+2)^2$

$C = \left(\frac{1}{2}4\right)^2 = 4$

$3x^2 + 24x + 48$

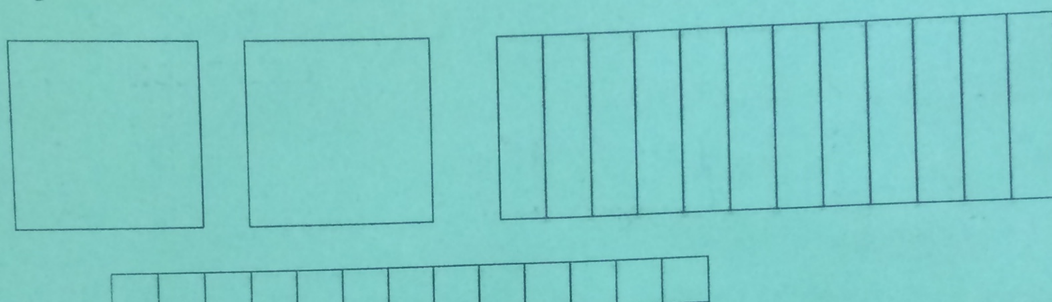
$3(x^2+8x+16)$

$3(x+4)^2$

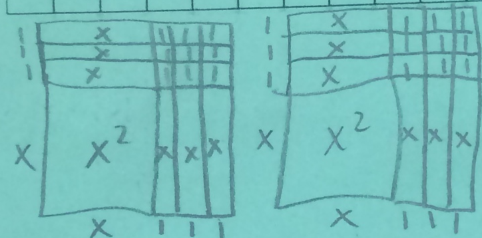
$C = \left(\frac{1}{2}8\right)^2 = 16$

Sometimes the quilt shop gets an order that turns out not to be more or less than a perfect square. Customer service always tries to fill orders with perfect squares, or at least, they start there and then adjust as needed.

6. Now here's a real mess! Customer service received an order for an area $A(x) = 2x^2 + 12x + 13$. Help them to figure out an equivalent expression for the area using the diagram.



Attempted perfect squares:

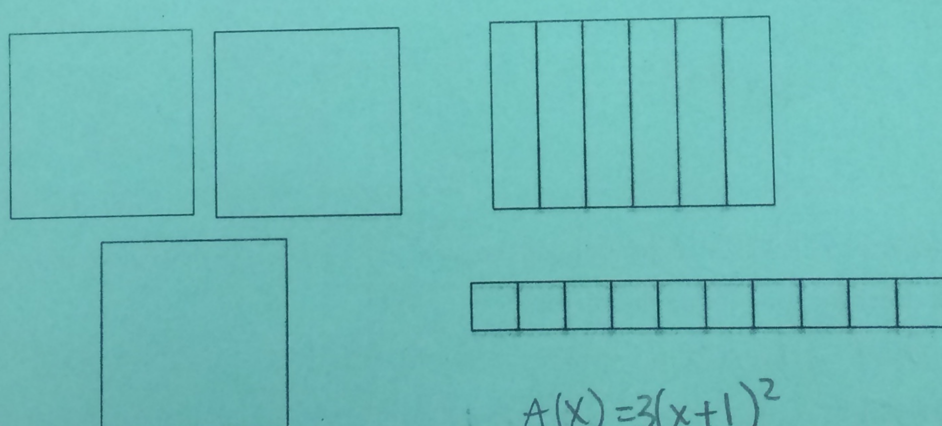


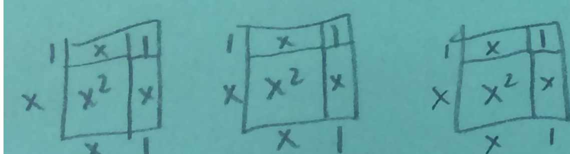
$A(x)=2(x^2+6x+9) \rightarrow 2(x^2+6x+9)-5$

$=2(x+3)^2 \rightarrow 2(x+3)^2-5$

The perfect squares requires 18 small squares but we only have 13.

7. Optima really needs to get organized. Here's another scrambled diagram. Write two equivalent equations for the area of this diagram:





$A(x)=3(x+1)^2$

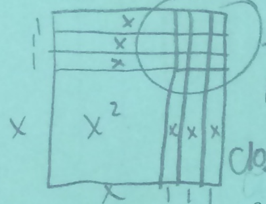
$A(x)=3(x^2+2x+1) \rightarrow A(x)=3(x^2+2x+1)+7$

$A(x)=3(x+1)^2+7$

The perfect squares require 3 small squares but we're given 10.

8. Optima realizes that not everyone is in need of perfect squares and not all orders are coming in as expressions that are perfect squares. Determine whether or not each expression below is a perfect square. Explain why the expression is or is not a perfect square. If it is not a perfect square, find the perfect square that seems "closest" to the given expression and show how the perfect square can be adjusted to be the given expression.

A. $A(x) = x^2 + 6x + 13$ ← not

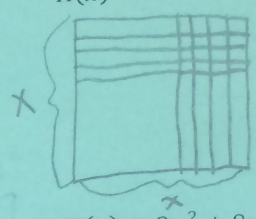


→ 9 squares

No, not a perfect square

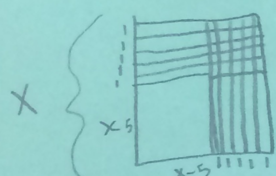
Closest square: $A(x) = (x+3)^2$
$= x^2 + 6x + 9$

B. $A(x) = x^2 - 8x + 16$



yes it's a perfect square

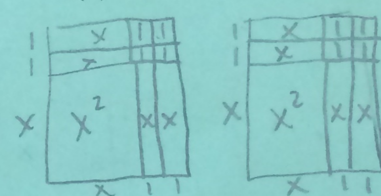
C. $A(x) = x^2 - 10x - 3$



No, not a perfect square

closest: $A(x) = x^2 - 10x + 25$

D. $A(x) = 2x^2 + 8x + 14$

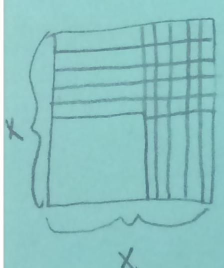


No not a perfect square,

$A(x) = 2x^2 + 8x + 8$
$= 2(x^2 + 4x + 4)$

E. $A(x) = 3x^2 - 30x + 75$

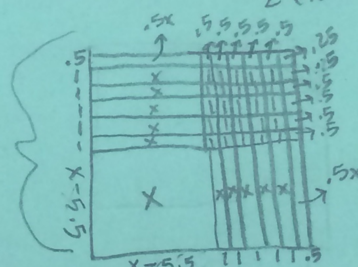
$3(x^2 - 10x + 25)$



Yes it's a perfect square

F. $A(x) = 2x^2 - 22x + 11$

$2(x^2 - 11x + \frac{11}{2})$



$A(x) = 2(x^2 + 11x + \frac{121}{4})$

No, There are $30.25 = \frac{121}{4}$ squares needed to fill the space needed to make a perfect square

9. Now let's generalize. Given an expression in the form $ax^2 + bx + c$ $(a \neq 0)$, describe a step-by-step process for completing the square.

1. take out (divide) the "a" from each term
2. determine the "c" value $(\frac{1}{2}b)^2$, to make a perfect square
3. adjust the expression by adding/subtracting a number on the end
4. write in vertex form $a(x-h)^2 + k$