

3.1 Experimenting with Exponents

A Develop Understanding Task

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Travis and Miriam are studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

1. Complete the following table and plot the data on the graph at the end of this task.

Hours into the study	0	1	2	3	4
Bacteria population (in thousands)	4	8	16	32	64

2. Write an equation for P , the population of the bacteria, as a function of time, t , and verify that it produces correct populations for $t = 1, 2, 3,$ and 4 hours.

$$P(t) = 4 \cdot 2^t$$

$\xrightarrow{\text{common factor}}$ $\xrightarrow{\text{starting value}}$

Exponential function

Travis and Miriam want to create a table with more entries; specifically, they want to fill in the population at each half hour. Unfortunately, they forgot to make these measurements so they decide to estimate the values.

Travis makes the following claim:

“If the population doubles in 1 hour, then half that growth occurs in the first half-hour and the other half in the second half-hour. So for example, we can find the population at $t = \frac{1}{2}$ by finding the average of the populations at $t = 0$ and $t = 1$.”

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

Hours into the study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria population (in thousands)	4	6	8	12	16	24	32	48	64

$$\frac{8+4}{2} = 6$$

$$\frac{8+16}{2} = 12$$

$$\frac{16+32}{2} = 24$$

$$\frac{32+64}{2} = 24$$

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4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Travis' table isn't multiplying by the same number everytime. His idea works for a linear function.

Miriam suggests they should fill in the data in the table in the following way:

"To make the estimates, I noticed that the population increases by the same factor each hour, and I think that this property should hold over each half-hour interval as well."

4. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor in order to produce consistent results. Use this constant multiplier to complete the table. Also plot Miriam's data on the graph at the end of this task.

Hours into the study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria population (in thousands)	4	5.66	8	11.31	16	22.63	32	45.25	64

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5. What if Miriam wanted to estimate the population every 20 minutes instead of every 30 minutes? What multiplier would she use for every third of an hour to be consistent with the population doubling every hour? Use this multiplier to complete the following table.

Hours into the study	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
Bacteria population (in thousands)	4	5.04	6.35	8	10.08	12.70	16	20.16	25.40	32

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- What number did you use as a multiplier to complete the table in problem 4?
- What number did you use as a multiplier to complete the table in problem 5?
- Give a detailed description of how you would estimate the population, P , at $t = \frac{5}{3}$ hours.

Plug in $t = \frac{5}{3} \rightarrow P\left(\frac{5}{3}\right) = 4 \cdot 2^{\frac{5}{3}} = 12.70$

or $10.08 \cdot \sqrt[3]{2} = 12.70$

