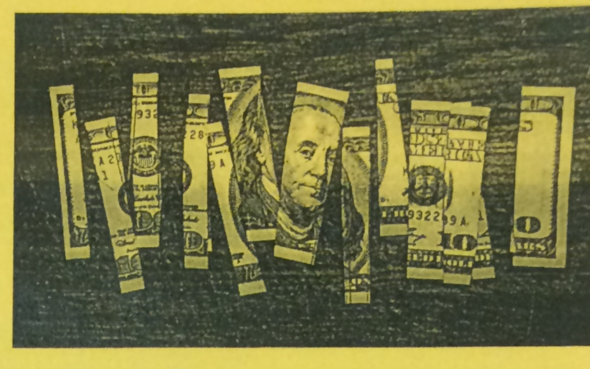
3.2 Half Interested

A Solidify Understanding Task



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Carlos and Clarita, the Martinez twins, have run a summer business every year for the past five years. Their first business, a neighborhood lemonade stand, earned a small profit that their father insisted they deposit in a savings account at the local bank. When the Martinez family moved a few months later, the twins decided to leave the money in the bank where it has been earning 5% interest annually. Carlos was reminded of the money when he found the annual bank statement they had received in the mail.

"Remember how Dad said we could withdraw this money from the bank when we are twenty years old," Carlos said to Clarita. "We have $382.88 in the account now. I wonder how much that will be five years from now?"

1. Given the facts listed above, how can the twins figure out how much the account will be worth five years from now when they are twenty years old? Describe your strategy and calculate the account balance.

Multiply each year by 1.05

keeps original value

5%

2. Carlos calculates the value of the account one year at a time. He has just finished calculating the value of the account for the first four years. Describe how he can find the next year's balance, and record that value in the table.

year	amount
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	488.66

$465.39 \cdot 1.05 = 488.66$

3. Clarita thinks Carlos is silly calculating the value of the account one year at a time, and says that he could have written a formula for the n^{th} year and then evaluated his formula when $n = 5$. Write Clarita's formula for the n^{th} year and use it to find the account balance at the end of year 5.

$A = 382.88(1.05)^t$

4. Carlos was surprised that Clarita's formula gave the same account balance as his year-by-year strategy. Explain, in a way that would convince Carlos, why this is so.

Carlos is multiplying by 1.05 over and over again. Clarita is using an exponent which is a short cut for repeated multiplication.

"I can't remember how much money we earned that summer," said Carlos. "I wonder if we can figure out how much we deposited in the account five years ago, knowing the account balance now?"

5. Carlos continued to use his strategy to extend his table year-by-year back five years. Explain what you think Carlos is doing to find his table values one year at a time, and continue filling in the table until you get to -5, which Carlos uses to represent "five years ago".

year	amount
-5	300.00
-4	315.00
-3	330.75
-2	347.29
-1	364.65
0	382.88
1	402.02
2	421.12
3	443.23
4	465.39
5	488.66

$\frac{364.65}{1.05} = 347.29$

$\frac{315.00}{1.05} = 300.00$

$\frac{347.29}{1.05} = 330.75$

$\frac{330.75}{1.05} = 315.00$

6. Clarita evaluated her formula for $n = -5$. Again Carlos is surprised that they get the same results. Explain why Clarita's method works.

$A = 382.88(1.05)^{-5} = 300.00$

negative exponents is a short cut for repeated division

Clarita doesn't think leaving the money in the bank for another five years is such a great idea, and suggests that they invest the money in their next summer business, *Curbside Rivalry* (which, for now, they are keeping top secret from everyone, including their friends). "We'll have some start up costs, and this will pay for them without having to withdraw money from our other accounts."

Carlos remarked, "But we'll be withdrawing our money halfway through the year. Do you think we'll lose out on this year's interest?"

"No, they'll pay us a half-year portion of our interest," replied Clarita.

"But how much will that be?" asked Carlos.

7. ~~Calculate the account balance and how much interest you think Carlos and Clarita should be paid if they withdraw their money ½ year from now. Remember that they currently have $382.88 in the account, and that they earn 5% annually. Describe your strategy.~~

Carlos used the following strategy: He calculated how much interest they should be paid for a full year, found half of that, and added that amount to the current account balance. 1 yr. increase = 402.02 − 382.88 = 19.14

19.14 ÷ 2 = 9.57 → 382.88 + 9.57 = 392.45

Clarita used this strategy: She substituted ½ for n in the formula $A = 382.88(1.05)^n$ and recorded this as the account balance. $A = 382.88(1.05)^{\frac{1}{2}} = 392.34$

8. This time Carlos and Clarita didn't get the same result. Whose method do you agree with and why?

Clarita's. Carlos is basing his strategy off a linear function and that doesn't work because it's exponential growth

Clarita is trying to convince Carlos that her method is correct. "Exponential rules are multiplicative, not additive. Look back at your table. We will earn $82.51 in interest during the next four years. If your method works we should be able to take half of that amount, add it to the amount we have now, and get the account balance we should have in two years, but it isn't the same."

9. Carry out the computations that Clarita suggested and compare the result for year 2 using this strategy as opposed to the strategy Carlos originally used to fill out the table.

82.51 ÷ 2 = 41.26 → 382.88 + 41.26 = 424.14 ≠ 422.12 (2 year amount

10. The points from Carlos' table (see question 2) have been plotted on the graph at the end of this task, along with Clarita's function. Plot the value you calculated in question 9 on this same graph. What does the graph reveal about the differences in Carlos' two strategies?

Carlos' strategy is above the actual price

11. Now plot Clarita's and Carlos' values for ½ year (see question 8) on this same graph.

plotted on graph

"Your data point seems to fit the shape of the graph better than mine," Carlos conceded, "but I don't understand how we can use ½ as an exponent. How does that find the correct factor we need to multiply by? In your formula, writing $(1.05)^5$ means multiply by 1.05 five times, and writing $(1.05)^{-5}$ means divide by 1.05 five times, but what does $(1.05)^{1/2}$ mean?"

Clarita wasn't quite sure how to answer Carlos' question, but she had some questions of her own. She decided to jot them down, including Carlos' question:

- What numerical amount do we multiply by when we use $(1.05)^{1/2}$ as a factor?
- What happens if we multiply by $(1.05)^{1/2}$ and then multiply the result by $(1.05)^{1/2}$ again? Shouldn't that be a full year's worth of interest? Is it?
- If multiplying by $(1.05)^{1/2} \cdot (1.05)^{1/2}$ is the same as multiplying by 1.05, what does that suggest about the value of $(1.05)^{1/2}$?

12. Answer each of Clarita's questions listed above as best as you can.

- $1.05^{\frac{1}{2}} \approx 1.004$
- $1.05^{\frac{1}{2}} \cdot 1.05^{\frac{1}{2}} = 1.05^1$, yes it does give full year's interest
- $1.05^{\frac{1}{2}} = \sqrt[2]{1.05}$ because $\sqrt[2]{1.05} \cdot \sqrt[2]{1.05}$ is also equal to 1.05

As Carlos is reflecting on this work, Clarita notices the date on the bank statement that started this whole conversation. "This bank statement is three months old!" she exclaims. "That means the bank will owe us ¾ of a year's interest."

"So how much interest will the bank owe us then?", asked Carlos.

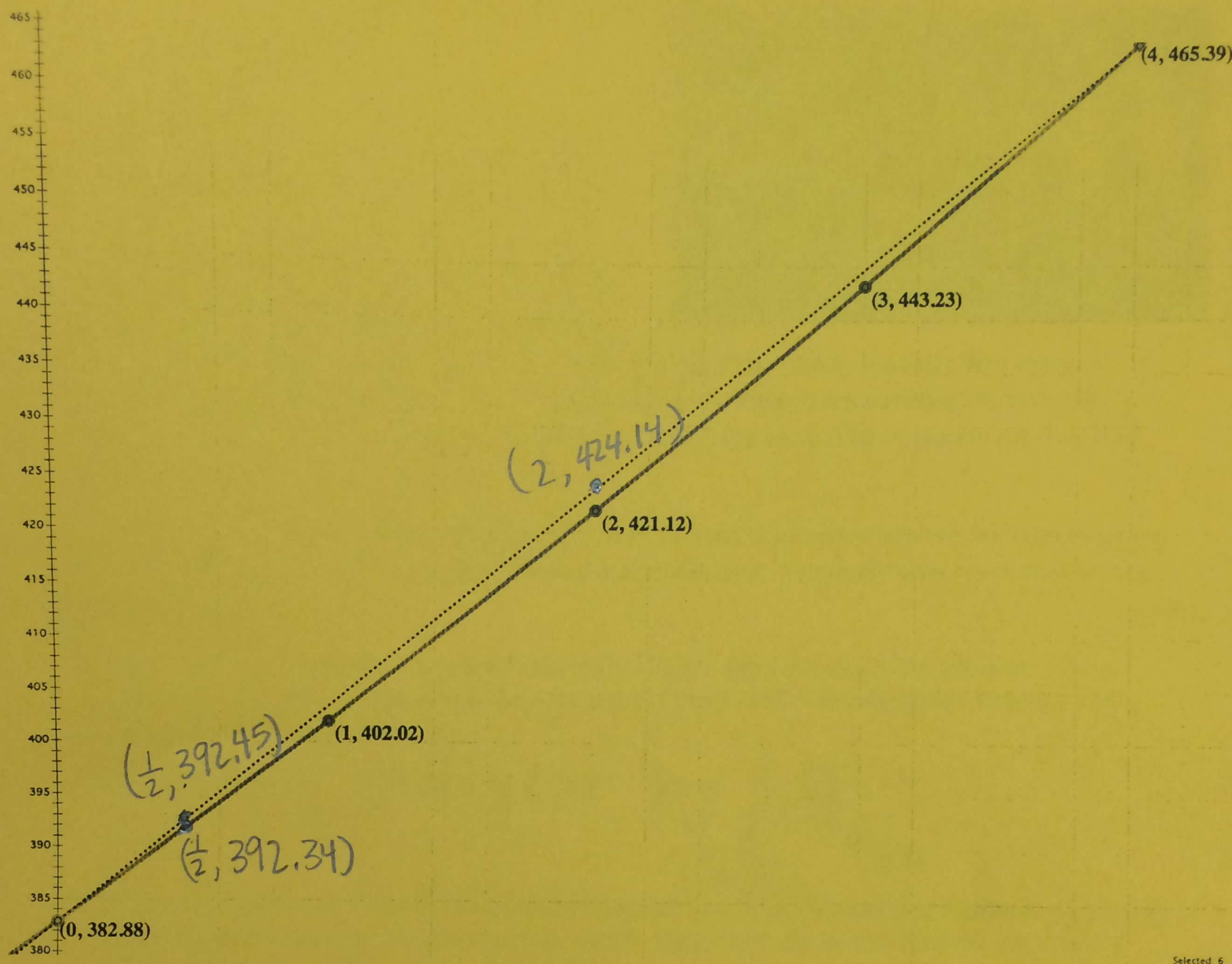
13. Find as many ways as you can to answer Carlos' question: How much will their account be worth in ¾ of a year (nine months) if it earns 5% annually and is currently worth $382.88?

$A = 382.88(1.05)^{\frac{3}{4}} = 397.15$

$A = 382.44 \cdot \sqrt[3]{1.05}^{\,4} = 397.15$

$A = 382.88 \cdot 1.05^{\frac{1}{4}} \cdot 1.05^{\frac{1}{4}} \cdot 1.05^{\frac{1}{4}} = 397.15$

$A = 382.88 \cdot (1.05^{\frac{1}{4}})^3 = 397.15$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(3, 443.23)
(2, 424.14)
(2, 421.12)
(1, 402.02)
(½, 392.45)
(½, 392.34)
(0, 382.88)
Selected 6