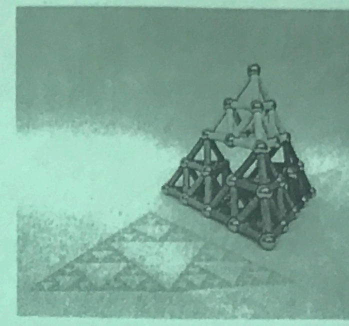
6.3 Similar Triangles and Other Figures

A Solidify Understanding Task



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Two figures are said to be *congruent* if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In Mathematics I we found that we only needed three pieces of information to guarantee that two triangles were congruent: SSS, ASA or SAS.

What about AAA? Are two triangles congruent if all three pairs of corresponding angles are congruent? In this task we will consider what is true about such triangles.

Part 1

So far similarity to us is two shapes that are the same just different sizes.

Definition of Similarity: Two figures are *similar* if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

Mason and Mia are testing out conjectures about similar polygons. Here is a list of their conjectures.

Conjecture 1: *All rectangles are similar.* F

Conjecture 2: *All equilateral triangles are similar.* T

Conjecture 3: *All isosceles triangles are similar.* F

Conjecture 4: *All rhombuses are similar.* F

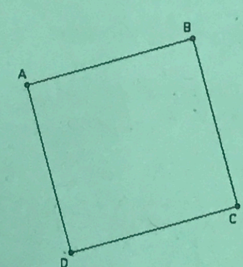
Conjecture 5: *All squares are similar.* T

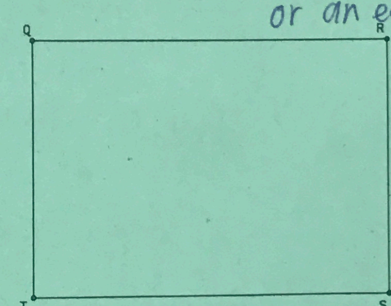
1. Which of these conjectures do you think are true? Why?

1,3,5 are false because not all of those shapes will be similar.
2,5 are ~~fals~~ true because no matter how big or small it gets it will still be a square or an equilateral triangle.

Mason is explaining to Mia why he thinks conjecture 1 is true using the diagram given below.

"All rectangles have four right angles. I can translate and rotate rectangle *ABCD* until vertex *A* coincides with vertex *Q* in rectangle *QRST*. Since ∠*A* and ∠*Q* are both right angles, side *AB* will lie on top of side *QR*, and side *AD* will lie on top of side *QT*. I can then dilate rectangle *ABCD* with point *A* as the center of dilation, until points *B*, *C*, and *D* coincide with points *R*, *S*, and *T*.

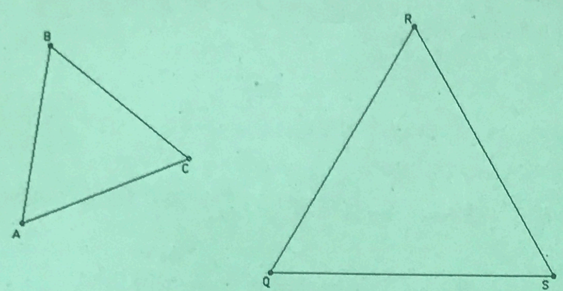




2. Does Mason's explanation convince you that rectangle *ABCD* is similar to rectangle *QRST* based on the definition of similarity given above? Does his explanation convince you that *all rectangles are similar*? Why or why not?

False, give explanation for why

Mia is explaining to Mason why she thinks conjecture 2 is true using the diagram given below.



"All equilateral triangles have three 60° angles. I can translate and rotate ΔABC until vertex A coincides with vertex Q in ΔQRS. Since $\angle A$ and $\angle Q$ are both 60° angles, side AB will lie on top of side QR, and side AC will lie on top of side QS. I can then dilate ΔABC with point A as the center of dilation, until points B and C coincide with points R and S."

3. Does Mia's explanation convince you that ΔABC is similar to ΔQRS based on the definition of similarity given above? Does her explanation convince you that *all equilateral triangles are similar*? Why or why not?

True, give explanation for why

4. For each of the other three conjectures, write an argument like Mason's and Mia's to convince someone that the conjecture is true, or explain why you think it is not always true.

   a. Conjecture 3: *All isosceles triangles are similar.*

   False, give explanation

   b. Conjecture 4: *All rhombuses are similar.*

   False, give explanation

   c. Conjecture 5: *All squares are similar.*

   True, give explanation

While the definition of similarity given at the beginning of the task works for all similar figures, an alternative definition of similarity can be given for polygons: **Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.** ← new similarity definition

5. How does this definition help you find the error in Mason's thinking about conjecture 1?

The sides of any two rectangles will not always be proportional, even though all angles are 90°.

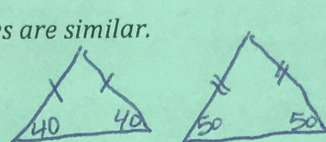
6. How does this definition help confirm Mia's thinking about conjecture 2?

All angles and sides in an equilateral triangle are the same, so any two will have all 60° angles, and sides can be scaled to match so they are proportional.

7. How might this definition help you think about the other three conjectures?

a. Conjecture 3: *All isosceles triangles are similar.*

False counterexample:



angles will not match up so these isosceles Δ's aren't similar

b. Conjecture 4: *All rhombuses are similar.*

False counterexample:

angles will not match up so these rhombuses will not be similar

c. Conjecture 5: *All squares are similar.*

True, since all squares have 90° angles and all 4 sides are the same, any square can be scaled to match up with any other square. The sides will always be proportional.

Part 2 (AAA Similarity)

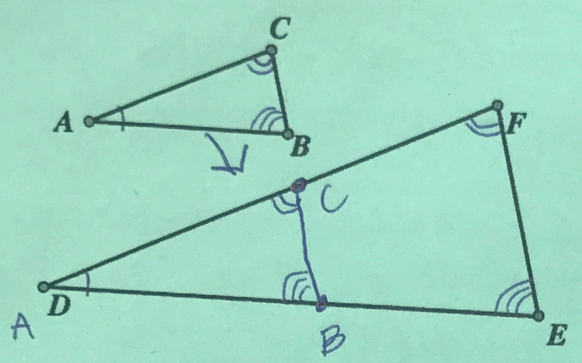
From our work above with rectangles it is obvious that knowing that all rectangles have four right angles (an example of AAAA for quadrilaterals) is not enough to claim that all rectangles are similar. What about triangles? In general, are two triangles similar if all three pairs of corresponding angles are congruent?

8. Decide if you think the following conjecture is true.

Conjecture: *Two triangles are similar if their corresponding angles are congruent.*

True

9. Explain why you think the conjecture—*two triangles are similar if their corresponding angles are congruent*—is true. Use the following diagram to support your reasoning, Remember to start by marking what you are given to be true (AAA) in the diagram.

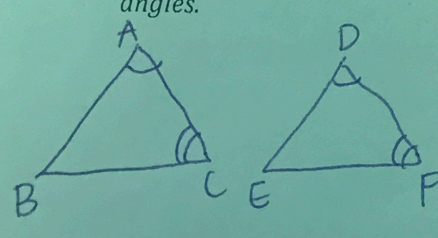


Hint: *If you translate A to D, where do points B and C end up?*

Since $\angle C \cong \angle F$ and $\angle B \cong \angle E$, $\overline{CB} \parallel \overline{FE}$. Therefore $\triangle ABC$ can be dilated using center D so that $\triangle ABC \cong \triangle DEF$. Thus $\triangle ABC \sim \triangle DEF$, by AA

10. Mia thinks the following conjecture is true. She calls it "*AA Similarity for Triangles.*" What do you think? Is it true? Why?

Conjecture: *Two triangles are similar if they have two pair of corresponding congruent angles.*



If $\angle A \cong \angle D$ and $\angle C \cong \angle F$ then $\angle B = 180-(m\angle A + m\angle C)$ and $m\angle E = 180-(m\angle D + m\angle F)$ so $\angle B \cong \angle E$. So by AAA, $\triangle ABC \sim \triangle DET$.