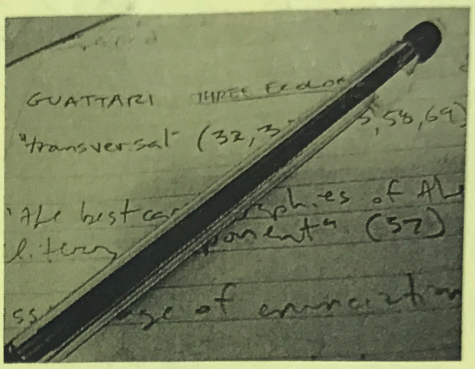
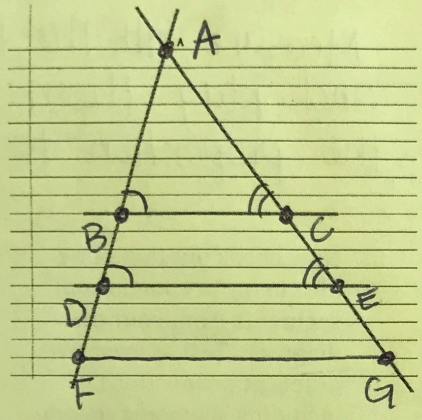
6.4 Cut By a Transversal

A Solidify Understanding Task

Draw two intersecting transversals on a sheet of lined paper, as in the following diagram. Label the point of intersection of the transversals *A*. Select any two of the horizontal lines to form the third side of two different triangles.



$\overline{BC} \parallel \overline{DE}$



1. What convinces you that the two triangles formed by the transversals and the horizontal lines are similar?

Because $\overline{BC} \parallel \overline{DE}$, $\angle B \cong \angle D$ and $\angle C \cong \angle E$ so by AA Similarity $\triangle ABC \sim \triangle ADE$.

2. Label the vertices of the triangles. Write some proportionality statements about the sides of the triangles and then verify the proportionality statements by measuring the sides of the triangles.

need 4 statements

$\frac{AB}{AD} = \frac{AC}{AE}$ (verify)

$\frac{AB}{AD} = \frac{BC}{DE}$ (verify)

$\frac{AC}{AE} = \frac{BC}{DE}$ (verify)

$\frac{AB}{AC} = \frac{AD}{AE}$ (verify)

3. Select a third horizontal line segment to form a third triangle that is similar to the other two. Write some additional proportionality statements and verify them with measurements.

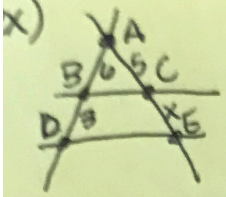
3 more statements

$\frac{AF}{AB} = \frac{AG}{AC}$ (verify)

$\frac{AG}{AC} = \frac{FG}{BC}$ (verify)

$\frac{AF}{AB} = \frac{FG}{BC}$ (verify)

x)



To solve for x:

$\frac{6}{3} = \frac{5}{x}$ or $\frac{6}{3} \times \frac{5}{x}$

$2 = 2$

$6x = 5 \cdot 3$

$\frac{6x}{6} = \frac{15}{6}$

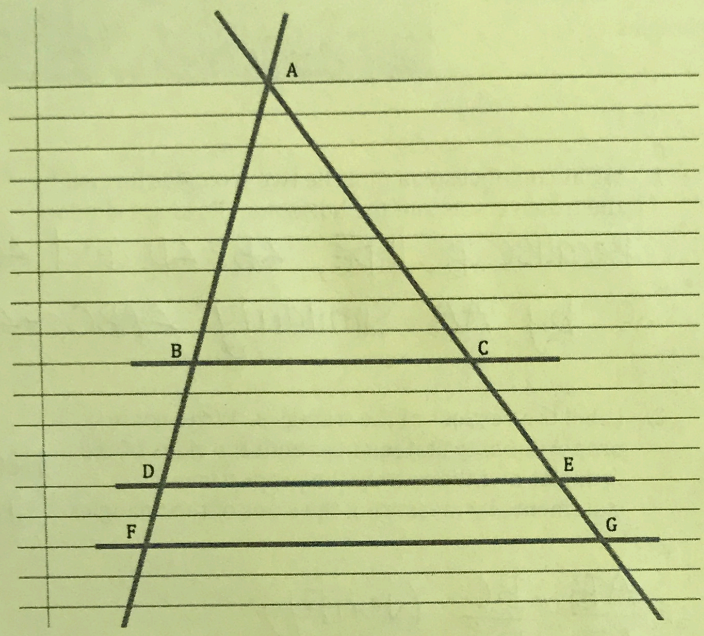
$x = 2.5$

$x = 2.5$

not full side lengths ↓ ↙ not full side lengths

Tristan has written this proportion for question 3, based on his diagram: $\frac{BD}{AB} = \frac{CE}{AC}$

Tia thinks Tristan's proportion is wrong, because some of the segments in his proportion are not sides of a triangle.



4. Check out Tristan's idea using measurements of the segments in his diagram at the left.

Measure all the lengths and plug them in to the proportion to verify.

5. Now check out this same idea using proportions of segments from your own diagram. Test at least two different proportions, including segments that do not have *A* as one of their endpoints.

$\frac{BD}{DF} = \frac{CE}{EG}$ (verify)

6. Based on your examples, do you think Tristan or Tia is correct?

Tristan, because parallel lines cut other lines proportionally

Tia still isn't convinced, since Tristan is basing his work on a single diagram. She decides to start with a proportion she knows is true: $\frac{AD}{AB} = \frac{AE}{AC}$. (Why is this true?) Because $\triangle ABC \sim \triangle ADE$

Tia realizes that she can rewrite this proportion as $\frac{AB + BD}{AB} = \frac{AC + CE}{AC}$ (Why is this true?) $AB+BD = AD$ and $AC+CE = AE$

Can you use Tia's proportion to prove algebraically that Tristan is correct?

$\frac{AB+BD}{AB} = \frac{AC+CE}{AC} \rightarrow \frac{AB}{AB} + \frac{BD}{AB} = \frac{AC}{AC} + \frac{CE}{AC} \rightarrow 1 + \frac{BD}{AB} = 1 + \frac{CE}{AC} \rightarrow \frac{BD}{AB} = \frac{CE}{AC}$ → same as Tristan's proportion above