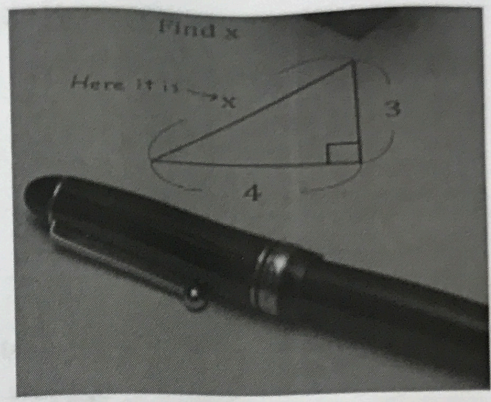


# 6.7 Pythagoras by Proportions

## A Practice Understanding Task

There are many different proofs of the Pythagorean Theorem. Here is one based on similar triangles.

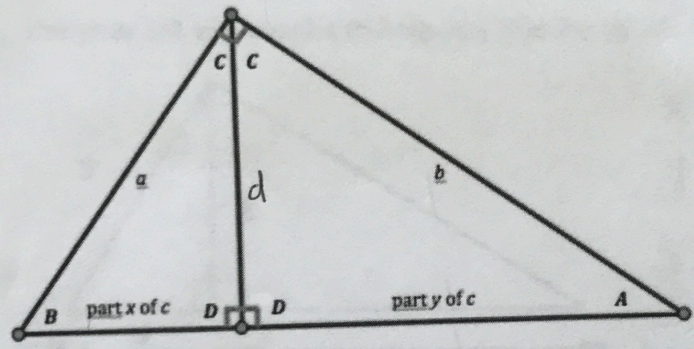
Step 1: Cut a 4 x 6 index card along one of its diagonals to form two congruent right triangles.



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Step 2: In each right triangle, draw an altitude from the right angle vertex to the hypotenuse.

Step 3: Label each triangle as shown in the following diagram. Flip each triangle over and label the matching sides and angles with the same names on the back as on the front.



Step 4: Cut one of the right triangles along the altitude to form two smaller right triangles.

Step 5: Arrange the three triangles in a way that convinces you that all three right triangles are similar. You may need to reflect and/or rotate one or more triangles to form this arrangement.

Step 6: Write proportionality statements to represent relationships between the labeled sides of the triangles.

~~$\frac{a}{b} = \frac{x}{y}$~~   ~~$\frac{a}{c} = \frac{d}{y}$~~   ~~$\frac{b}{c} = \frac{d}{x}$~~   $\frac{a}{c} = \frac{x}{a}$   $\frac{c}{b} = \frac{b}{y}$  need 3 total

Step 7: Solve one of your proportions for x and the other proportion for y. (If you have not written proportions that involve x and y, study your set of triangles until you can do so.)

~~$x = \frac{ay}{b}$~~   ~~$y = \frac{bx}{a}$~~

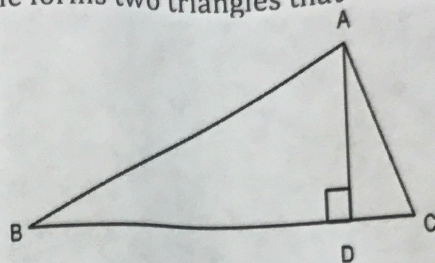
Step 8: Work with the equations you wrote in step 7 until you can show algebraically that  $a^2 + b^2 = c^2$ . (Remember,  $x + y = c$ .)

$x = \frac{a^2}{c}$   $y = \frac{b^2}{c}$

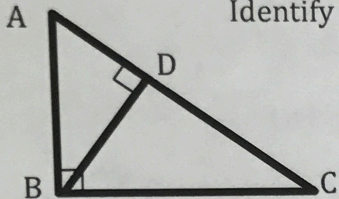
$$x + y = \frac{a^2}{c} + \frac{b^2}{c} = \frac{a^2 + b^2}{c}$$
$$\downarrow$$
$$c \cdot c = \left(\frac{a^2}{c} + \frac{b^2}{c}\right) c$$
$$c^2 = a^2 + b^2$$



**Theorem:** The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

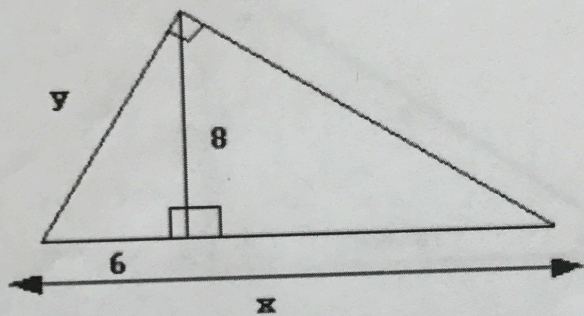


Identify the similar triangles in the figure to the left.



$\triangle ABC \sim \triangle ADB \sim \triangle BDC$

Use your set of triangles to help you find the values of  $x$  and  $y$  in the following diagram.

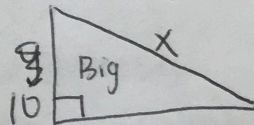
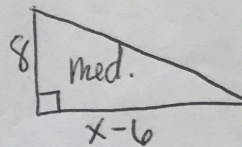
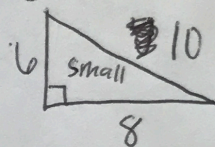


$$\frac{x}{y} = \frac{y}{6}$$

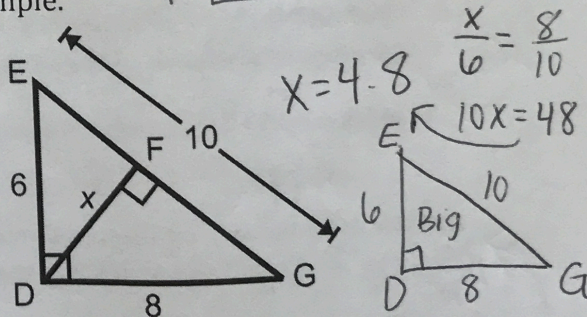
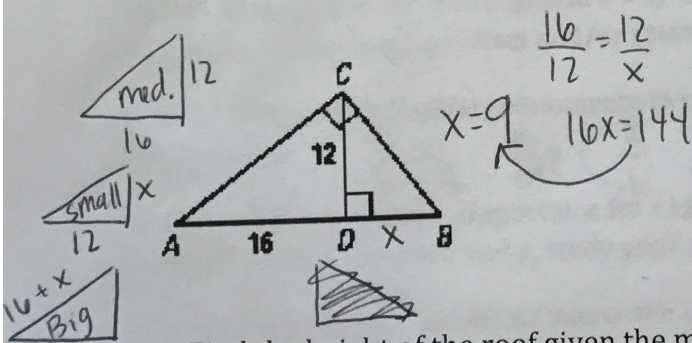
$$\frac{6}{10} = \frac{10}{x}$$

$$6x = 100$$

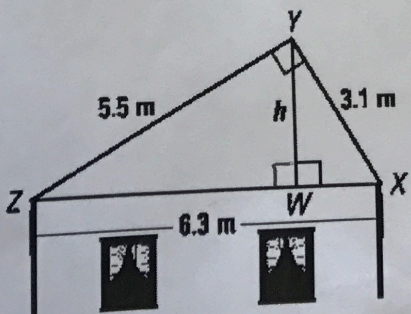
$$x = 16.67$$



Find the measure(s) of the missing side(s) in each example.



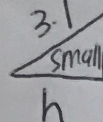
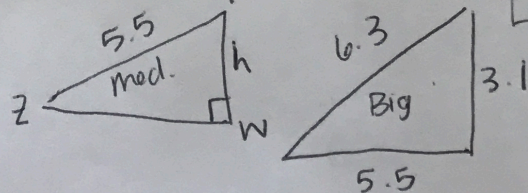
Find the height of the roof given the measurements on the figure below.



$h = 2.7$

$$\frac{h}{3.1} = \frac{5.5}{6.3}$$

$$6.3h = 17.05$$



$$\frac{6.3}{3.1} = \frac{5.5}{h}$$