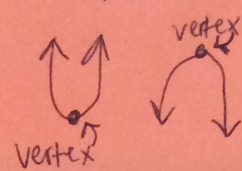
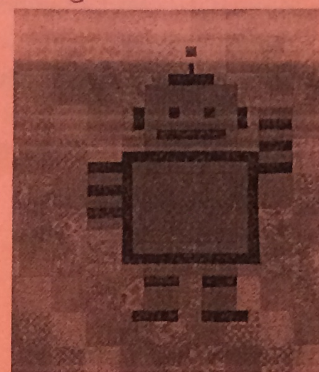
Mrs. Clark's Notes



vertex: turning point of graph, the max or min value on the graph.



2.1 Transformers: Shifty y's

A Develop Understanding Task

Optima is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x units (which can be inches or centimeters) is modeled by the function, $A(x) = x^2$ square units.

1. What is the domain of the function $A(x)$ in this context?

   Domain is the possible side lengths: $(0, \infty)$. We can't have side length less than 0, and a side length of 0 means we don't have a square. There isn't a limit to how big the square could be so we say ∞ is our max.

2. Match each statement about the area to the function that models it:

Matching Equation (A,B, C, or D)	Statement	Function Equation
B	The length of each side is increased by 5 units.	A) $A = 5x^2$
C	The length of each side is multiplied by 5 units.	B) $A = (x + 5)^2$
D	The area of a square is increased by 5 square units.	C) $A = (5x)^2$
A	The area of a square is multiplied by 5.	D) $A = x^2 + 5$

Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers) $(-\infty, \infty)$ $\mathbb{R}$ and wondering about how changes to the equation of the function like adding 5 or multiplying by 5 affect the graph. She decided to make predictions about the effects and then check them out.

3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	Differences from the graph of $y = x^2$
$y = 5x^2$	still a parabola, ~~different shape~~	stretched up, skinnier parabola
$y = (x + 5)^2$	still a parabola, same size	slid to the left 5 units
$y = (5x)^2$	Still a parabola,	stretched really skinny y-values went up really fast
$y = x^2 + 5$	still a parabola, same size	slid up the y-axis 5 units

dilation: stretched or shrunk away from point

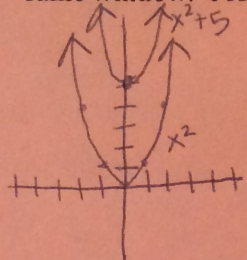
rotation: rotated around a point

mathematics vision project

translation: slides doesn't change shape

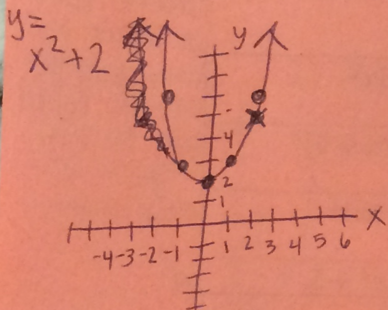
reflection: reflects over line "mirror image"

4. Optima decides to test her ideas using technology. She thinks that it is always a good idea to start simple, so she decides to go with $y = x^2 + 5$. She graphs it along with $y = x^2$ in the same window. Test it yourself and describe what you find.

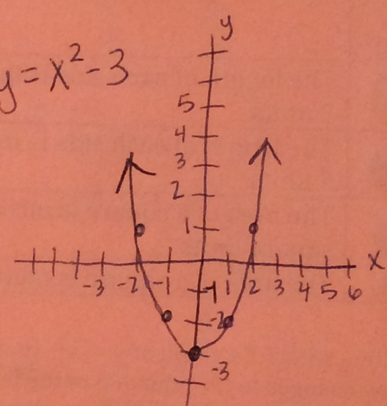


The graph shifted up the y-axis 5 units.

5. Knowing that things make a lot more sense with more representations, Optima tries a few more examples like $y = x^2 + 2$ and $y = x^2 - 3$, looking at both a table and a graph for each. What conclusion would you draw about the effect of adding or subtracting a number to $y = x^2$? Carefully record the tables and graphs of these examples On graph paper and explain why your conclusion would be true for any value of k, given, $y = x^2 + k$.



x	x^2+2
-2	6
-1	3
0	2
1	3
2	6



x	x^2-3
-2	1
-1	-2
0	-3
1	-2
-2	1

Adding a "K" value to the end of $y = x^2$, moves the graph vertically along the y-axis.

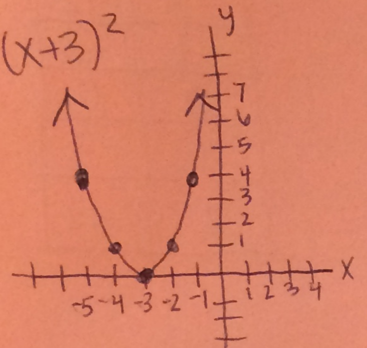
If "K" is positive, it moves the graph <u>up</u> k units.

If "K" is negative, it moves the graph <u>down</u> k units.

6. After her amazing success with addition in the last problem, Optima decides to look at what happens with addition and subtraction inside the parentheses, or as she says it, "adding to the x before it gets squared". Using your technology, decide the effect of h in the equations: $y = (x + h)^2$ and $y = (x - h)^2$. (Choose some specific numbers for h.) Record a few examples (both tables and graphs) On graph paper and explain why this effect on the graph occurs.

$h = 3$

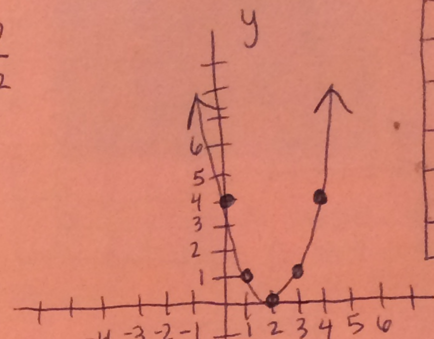
$y = (x+3)^2$



x	$(x+3)^2$
-5	4
-4	1
-3	0
-2	1
-1	4

$h = -2$

$y = (x-2)^2$



x	$(x-2)^2$
0	4
1	1
2	0
3	1
4	4

$y=(x+h)^2$ moves the graph horizontally along the x-axis to the left h units.

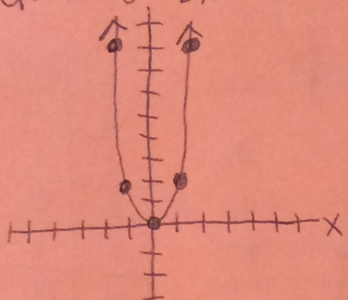
$y=(x-h)^2$ moves the graph horizontally along the x-axis to the right h units.

7. Optima thought that #6 was very tricky and hoped that multiplication was going to be more straightforward. She decides to start simple and multiply by -1, so she begins with $y = -x^2$. Predict what the effect is on the graph and then test it. Why does it have this effect?

Multiplying by -1 reflects the graph over the x-axis so that the parabola is now upside down.

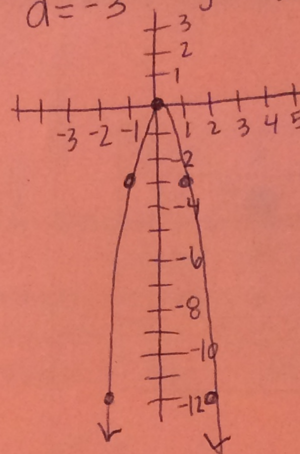
8. Optima is encouraged because that one was easy. She decides to end her investigation for the day by determining the effect of a multiplier, a, in the equation: $y = ax^2$. Using both positive and negative numbers, fractions and integers, create at least 4 tables and matching graphs to determine the effect of a multiplier.

$a = 2$ $y = 2x^2$



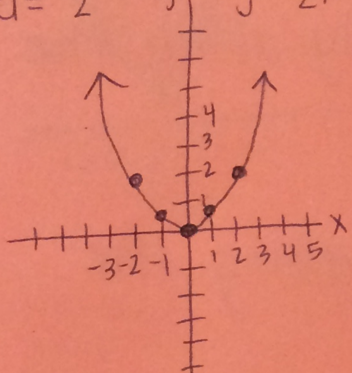
x	$2x^2$
-2	8
-1	2
0	0
1	2
2	8

$a = -3$ $y = -3x^2$



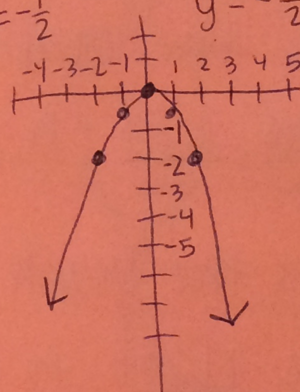
x	$-3x^2$
-2	-12
-1	-3
0	0
1	-3
2	-12

$a = \frac{1}{2}$ $y = \frac{1}{2}x^2$



x	$\frac{1}{2}x^2$
-2	2
-1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	2

$a = -\frac{1}{2}$ $y = -\frac{1}{2}x^2$



x	$-\frac{1}{2}x^2$
-2	-2
-1	$-\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	-2

When $a>1$, the graph is stretched upwards. The y-values go up really fast.

When $0<a<1$, the graph is stretched outward. The y-values go up half as fast.

When $a<-1$, the graph is reflected downward. The y-values go down really fast.

When $-1<a<0$, the graph is reflected & stretched outward. y-values go down half as fast.