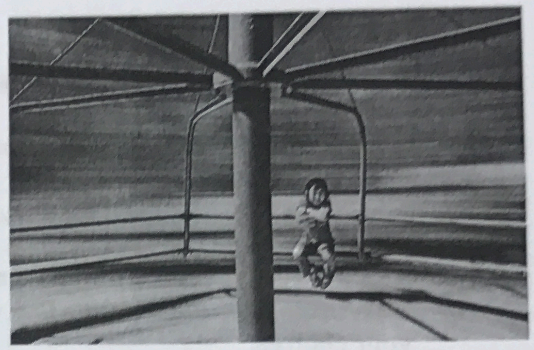


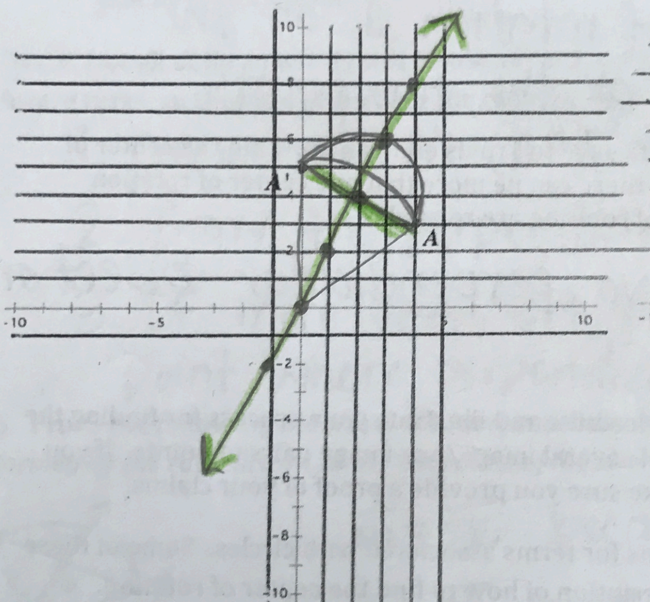
7.1 Centered

A Develop Understanding Task

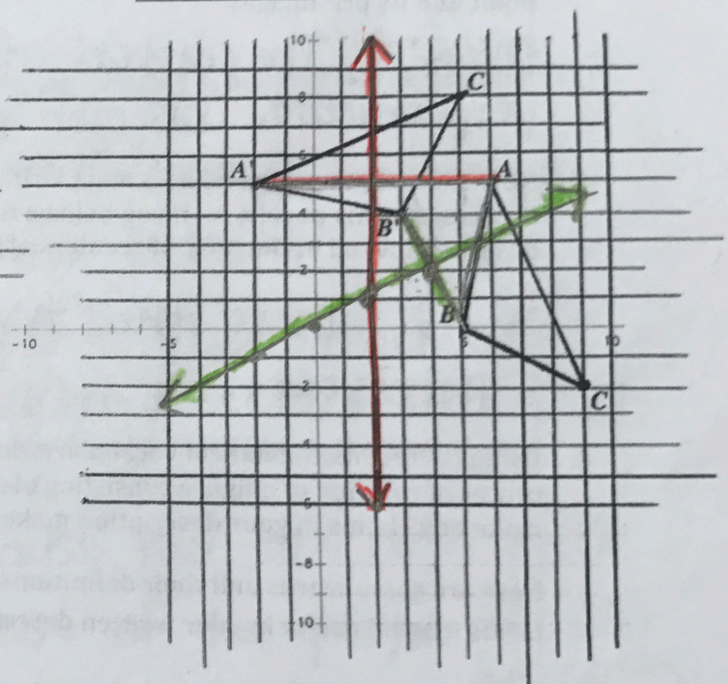
Travis and Tehani know how to construct the image of a rotation when given the center and angle of rotation, but today they have encountered a different issue: how do you find the center of rotation when a rotated image and its pre-image are given? They decide to explore this idea with their friends, Carlos and Clarita.

Each pair of friends creates a "puzzle" for the other pair by sketching a drawing on graph paper in which a rotation of a figure is shown, but the center of rotation is not marked. The other pair has to figure out where the center of rotation is located. Here are the "puzzles" they created for each other.

Travis and Tehani's Puzzle



Carlos and Clarita's Puzzle



Carlos and Clarita think that the puzzle they have been given is too easy, since it only consists of a single rotated point and its pre-image.

Carlos: "The center of rotation is at the midpoint (2,4), halfway between the image and pre-image points, and the point has been rotated 180°."

Clarita disagrees: "The center of rotation is at the point (0, 0) since both the image and pre-image points are 5 units away from origin. I'll need to use a protractor to find the angle of rotation."

Laughing, Tehani says, "You're both wrong. We didn't use either (2, 4) or (0, 0) as the center of rotation when we created the puzzle."

Carlos replies, "I can see how *both* of our points can be the center of rotation, but now I think that with a single image/pre-image pair of points *any* point can be the center of rotation."

1. This puzzle has turned out to be more challenging than Carlos and Clarita thought. List at least three additional points that could be considered as the center of rotation, and justify your choices.

   (1,2), (3,6), (4,8), (-1,-2)

2. What do you think about Carlos' last statement, "Any point can be the center of rotation"? Do you agree or disagree? If you agree, explain why any point works as the center of rotation for a single rotated point. If you disagree, what would be a better statement to make about the set of points that can be used as the center of rotation for a single rotated point and its pre-image?

   Disagree, The centers of rotation lie on the perpendicular bisector of the two points.

3. Now examine the puzzle Carlos and Clarita gave to Travis and Tehani. Find the center of rotation for this puzzle; or, if you believe there can be more that one center of rotation, describe how all of the possible centers of rotation are related.

   (2,1), where the two perpendicular bisectors intersect

4. Using correct mathematical vocabulary, describe and illustrate your process for finding the center of rotation of a figure consisting of several image/pre-image pairs of points. If you make any claims in your description make sure you provide a proof of your claims.

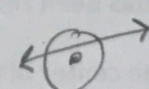
   Here are some words and their definitions for terms associated with circles. Some of these terms may be useful in your written description of how to find the center of rotation.

Circle—the set of all points in a plane equidistant from a fixed point called the center of the circle.

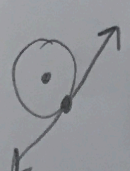
Concentric circles—a set of different circles that share the same center.

Chord—a line segment whose endpoints lie on a circle.

Secant—a line that intersects a circle at exactly two points.



Tangent—a line that intersects a circle at exactly one point.



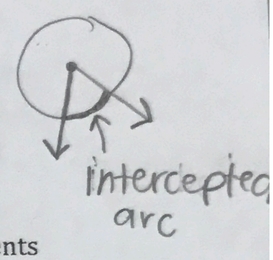
Diameter—a chord that passes through the center of a circle.

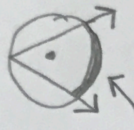
Radius—a line segment with one endpoint at the center of a circle and the other endpoint on the circle.

Note: the words *radius* and *diameter* also are used to refer to the lengths of these segments.

Arc—a portion of a circle.

Central angle—an angle whose vertex is at the center of a circle and whose sides pass through a pair of points on the circle.





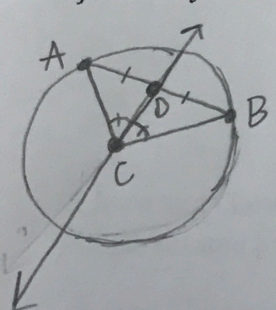
Inscribed angle—an angle formed when two secant lines, or a secant and tangent line, intersect at a point on a circle.

Intercepted arc—the portion of a circle that lies between two lines, rays or line segments that intersect the circle.

(Note: Not all of these words will be useful for answering question 4, but they will be useful in future tasks, so they are given here for reference.)

1. Connect $A \to A'$ ($B \to B'$ if necessary)
2. Draw the perpendicular bisector
3. If more than one point, then find the point where perpendicular bisectors' intersect.

5. Prove the following theorem: *The perpendicular bisector of a chord bisects the central angle formed by the radii drawn to the endpoints of the chord.*



1. $\overline{AB} = \overline{BC}$ because they are both radii.
2. $\overline{AD} = \overline{DB}$ because of the perpendicular bisector.
3. $\overline{DC} = \overline{DC}$ because it's the same line.
4. By SSS, $\triangle ACD \cong \triangle BCD$.
5. Thus $\angle ACD \cong \angle BCD$.