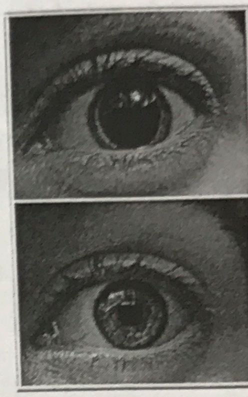
7.2 Circle Dilations

A Develop Understanding Task

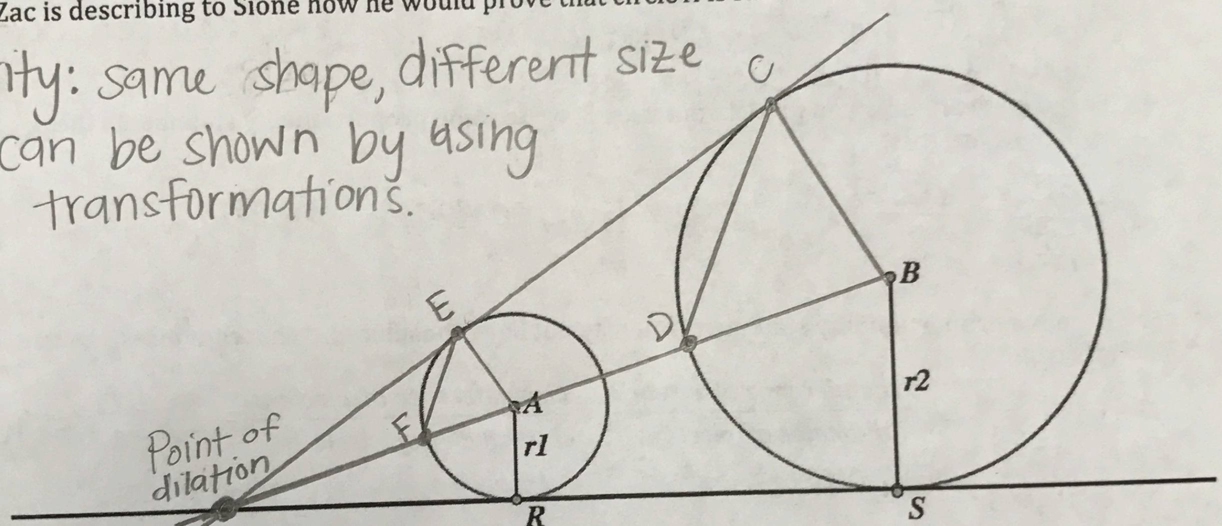


The statement "*all circles are similar*" may seem intuitively obvious, since all circles have the same shape even though they may be different sizes. However, we can learn a lot about the properties of circles by working on the proof of this statement.

Remember that the definition of similarity requires us to find a sequence of dilations and rigid motion transformations that superimposes one figure onto the other.

Zac is describing to Sione how he would prove that circle *A* is similar to circle *B*.

similarity: same shape, different size
can be shown by using
transformations.



Zac: "Translate circle *A* until its center coincides with the center of circle *B*. Then enlarge circle *A* by dilation until the points on circle *A* coincide with the points on circle *B*. Or, you could shrink circle *B* by dilation until the points on circle *B* coincide with the points on circle *A*."

Sione has some questions: "After the translation, what is the scale factor for the enlargement that carries circle *A* onto circle *B*? And, what is the scale factor for the reduction that carries circle *B* onto circle *A*?

1. How would you answer Sione's questions?

$A \to B$: scale factor $= \frac{r2}{r1}$ $\quad$ $B \to A$: scale factor $= \frac{r1}{r2}$

~ means similar

Based on Zac and Sione's discussion, we are probably convinced that circle *A* and circle *B* are similar. Another way we might convince ourselves that the two circles are similar would be to find the center of dilation that maps pre-image points from circle *A* onto corresponding image points on circle *B.*

2. Locate the center of dilation that carries circle *A* onto circle *B.* Explain how you know the point you found is the center of dilation. (Note that both circles have been drawn tangent to $\overrightarrow{RS}$.)

Intersection of the lines connecting the points

3. Draw some chords, triangles or other polygons in each circle that would be similar to each other. Explain how you know these corresponding figures are similar.

$\triangle AEF \sim \triangle BCD$, if you translates $\triangle AEF$ so that point F coincides with point D then dilate $\triangle AEF$, the triangle would be congruent.

4. Based on the figures you drew in question 3, write some proportionality statements that you know are true.

$\frac{\overline{BC}}{\overline{AE}} = \frac{r2}{r1}$ because they're both radii

5. Here is a proportionality statement you may not have considered. What convinces you that it is true?

$$\frac{\textit{circumference of circle A}}{\textit{diameter of circle A}} = \frac{\textit{circumference of circle B}}{\textit{diameter of circle B}} = \frac{\pi \cdot d}{d} = \pi \cdot 1 = \pi$$

For any size circles this statement is true.

Since this ratio of circumference to diameter is the same scale factor for all circles, this ratio has been given the name π (pi).

6. How much larger is the circumference of circle *B* than the circumference of circle *A*?

$\frac{(r2 - r1)}{\pi}$

7. Do you think the following proportion is true or false? Why?

$$\boxed{\frac{\textit{area of circle B}}{\textit{area of circle A}} = \frac{\textit{circumference of circle B}}{\textit{circumference of circle A}}} \rightarrow \frac{\pi (r2)^2}{\pi (r1)^2} \neq \frac{2\pi (r2)}{2\pi (r1)}$$

False, $\frac{(r2)^2}{(r1)^2} \neq \frac{(r2)}{(r1)}$.