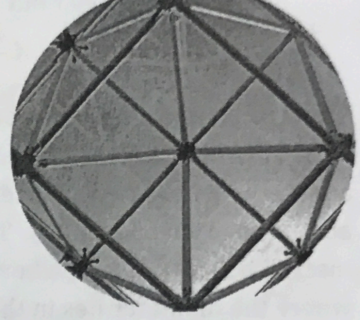
7.3 Cyclic Polygons

A Solidify Understanding Task



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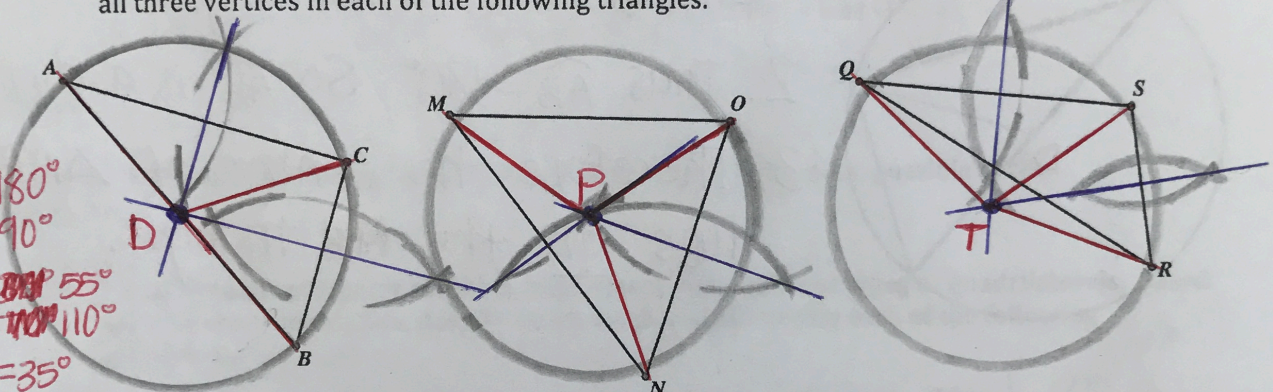
By definition, a cyclic polygon is a polygon that can be inscribed in a circle. That is, all of the vertices of the polygon lie on the same circle.

Part 1

In task *5.8 Centers of a Triangle* your work on Kara's notes and diagram should have convinced you that it is possible to locate a point that is equidistant from all three vertices of any triangle, and therefore all triangles are cyclic polygons. Use perpendicular bisectors of each side to find center

1. Based on Kara's work, use a compass and straightedge to construct the circles that contain all three vertices in each of the following triangles.



$\angle ADB = 180°$
$\angle ACB = 90°$
$\angle ABC = 55°$
$\angle ADC = 110°$
$\angle BDC = 35°$
$\angle BAC = 70°$

Since each vertex of an inscribed triangle lies on the circle, each angle of the triangle is an inscribed angle. We know that the sum of the measures of the interior angles of the triangle is 180° and that the sum of the measures of the three intercepted arcs is 360°.

2. Using one of the diagrams of an inscribed triangle you created above, illustrate and explain why this last statement is true.

$\angle ADB + \angle ADC + \angle BDC = 360°$ $\quad \angle ACB + \angle ABC + \angle BAC = 180°$

We know that the degree measure of an arc is, by definition, the same as the measure of the central angle formed by the radii that contain the endpoints of the arc. But how is the measure of an inscribed angle that intercepts this same arc related to the measure of the central angle and the intercepted arc? That is something useful to find out.

3. Using a protractor, find the measure of each arc represented on each circle diagram above. Then find the measure of each corresponding inscribed angle. Make a conjecture based on this data.

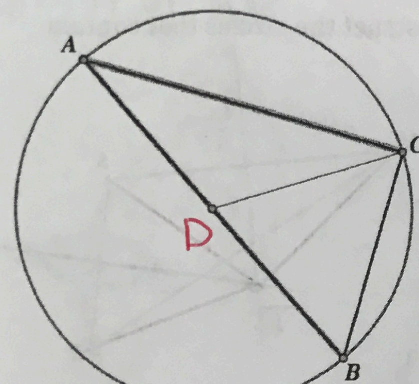
Shown above

My conjecture about the measure of an inscribed angle:

The measure of the inscribed angle is $\frac{1}{2}$ the measure of the central angle.

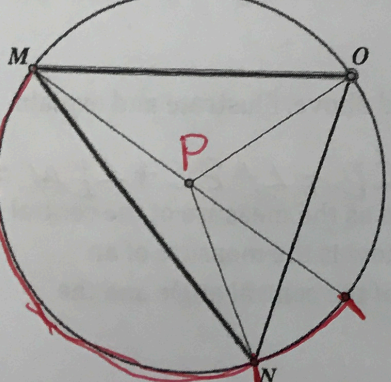
The three circle diagrams you created above have been reproduced below. One inscribed angle has been bolded in each triangle. A diameter of the circle has also been added to each diagram as an auxiliary line segment, as well as some additional line segments that will assist in writing proofs about the inscribed angles. Three cases are illustrated: case 1, where the diameter is a side of the inscribed angle; case 2, where the diameter lies in the interior of the inscribed angle; and case 3, where the diameter lies in the exterior of the inscribed angle. In each diagram, prove your conjecture for the inscribed angle shown in bold.

Case 1:



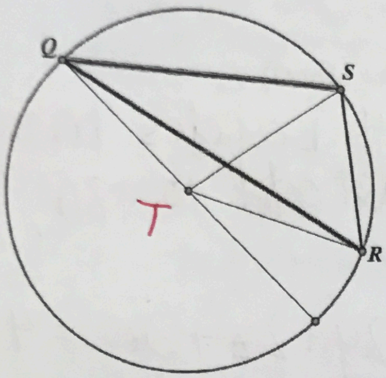
1. The measure of $\angle ADB = 180°$ and $\angle ADB$ is a central angle.
2. Thus $\widehat{AB} = 180°$. So $\overline{AB}$ is a diameter.
3. Therefore, the center of $\triangle ABC$ lies on the ~~[illegible]~~ triangle.

Case 2:



1. The measure of any angle in $\triangle MNO$ is less than 90°.
2. So the measure of any central angle through the vertices is less than 180°.
3. Therefore, the center of $\triangle MNO$ lies inside the triangle.

Case 3:



1. The measure of $\angle QSR$ is greater than 90°.

2. So the measure of $\angle QTR$ is greater than 180°, thus $\widehat{QR}$ is greater than 180°.

3. Therefore the center of $\triangle QSR$ is outside the triangle.

Part 2

We have found that all triangles are cyclic polygons. Now let's examine possible cyclic quadrilaterals.

4. Using dynamic geometry software, experiment with different types of quadrilaterals. Based on your experimentation, deicide which word best completes each of the following statements:

   a. [Some, all, no] squares are cyclic. → all sides are equal, 90° angles

   b. [Some, all, no] rhombuses are cyclic. → all sides are equal

   c. [Some, all, no] trapezoids are cyclic. → only 1 set of parallel lines

   d. [Some, all, no] rectangles are cyclic. → all 90° angles, 2 sets of ∥ sides

   e. [Some, all, no] parallelograms are cyclic. → 2 sets of parallel sides

Obviously, some generic quadrilaterals are cyclic, since you can select any four points on a circle as the vertices of a quadrilateral.

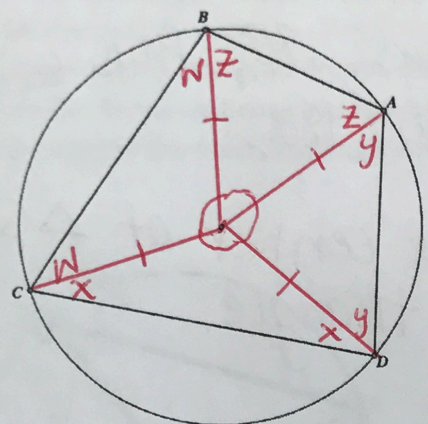
5. Using dynamic geometry software, experiment with cyclic quadrilaterals that are not parallelograms or trapezoids. Focus on the measurements of the angles. Make a conjecture about the measures of the angles of a cyclic quadrilateral. Then prove your conjecture using what you know about inscribed angles.

My conjecture about the angles of a cyclic quadrilateral:

In a cyclic quadrilateral, opposite angles add to 180°.

Proof of my conjecture:

(How might you use the following diagram to assist you in your proof?)



1. We have 4 isosceles triangles whose angle sums add to 180°.

2. $2w+2x+2y+2z+360° = 4 \cdot 180°$

3. $2w+2x+2y+2z+360° = 720°$
$\quad -360° \quad -360$

$$\frac{2w}{2}+\frac{2x}{2}+\frac{2y}{2}+\frac{2z}{2}=\frac{360}{2}$$

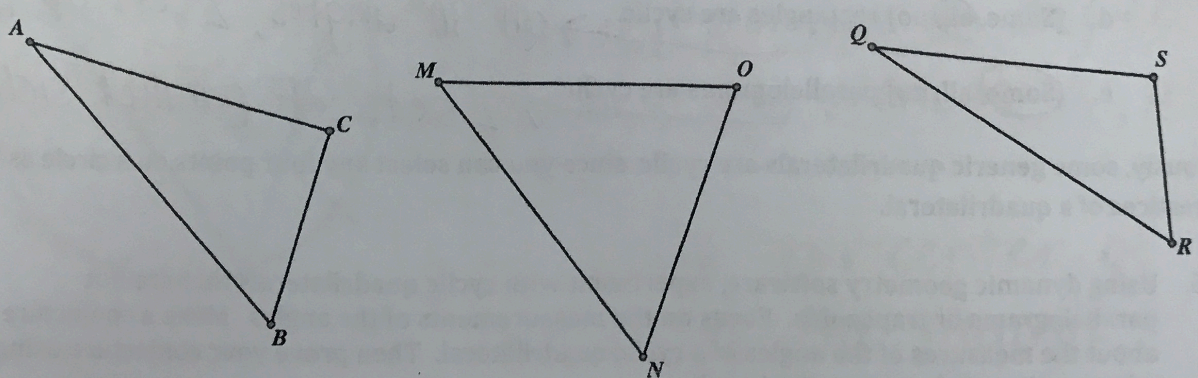
$(w+x)+(y+z)=180°$ or $(w+z)+(x+y)=180$

$\angle C + \angle A = 180°$ $\quad$ $\angle B + \angle D = 180°$

Part 3

In task *5.8 Centers of a Triangle,* your work on Kolton's notes and diagram should have convinced you that it is possible to locate a point that is equidistant from all three sides of a triangle, and therefore a circle can be inscribed inside every triangle.

6. Based on Kolton's work, use a compass and straightedge to construct the circles that can be inscribed in each of the following triangles.

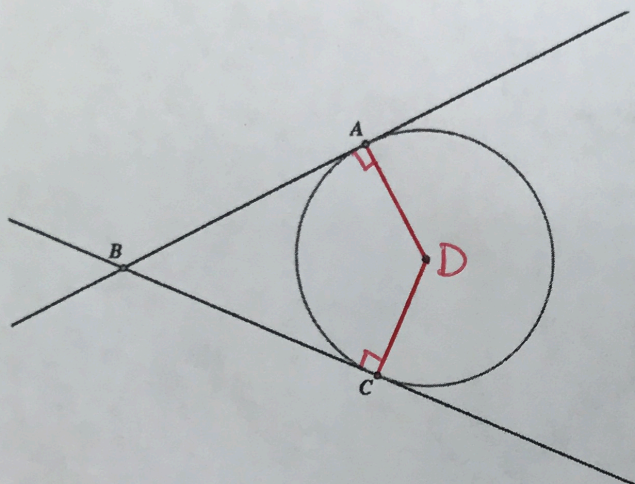


7. The angles of the triangle that are formed by the lines that are tangent to the circle are called circumscribed angles. Use dynamic geometry software to experiment with the measures of circumscribed angles relative to the arcs they intercept. Make a conjecture about the measures of the circumscribed angles. Then prove your conjecture using what you know about inscribed angles.

My conjecture about the measures of circumscribed angles:

The measure of the circumscribe angle and the central angle add to 180°.

Proof of my conjecture:



$\angle B + \angle D = 180^\circ$

8. Based on your work in this task and the previous task, describe a procedure for constructing a tangent line to a circle through a given point outside the circle.

1. Draw point away from circle.
2. construct a line that goes through the point and touches the circle.