

## 7.8 Madison's Round Garden

### A Practice and Develop Understanding Task

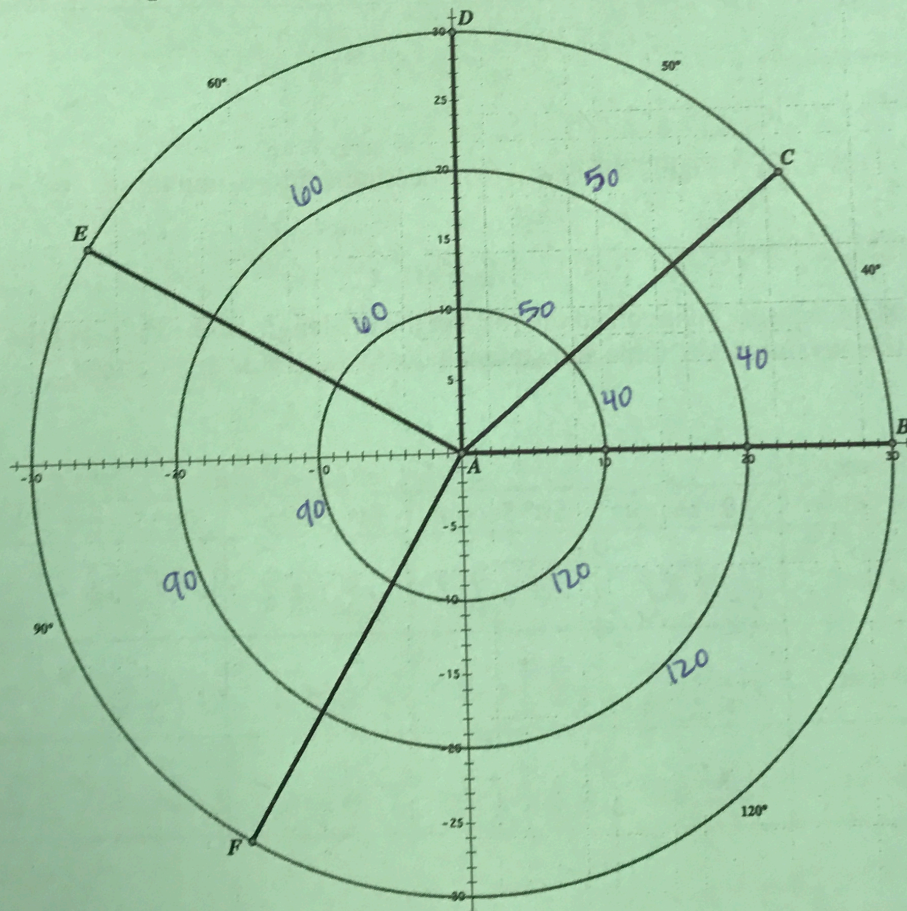
Last year Madison won the city's "Most Outstanding Garden" Award for her square garden. This year she plans to top that with her design for a beautiful round garden.



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Madison's design starts with a sprinkler in the center, and concentric rings of colorful flowers surrounding the central sprinkler. Pavers will create both circular pathways and pathways that look like spokes on a wheel between the flowers. The sprinkler can be adjusted so it waters just the inner circle of flowers, or it can be adjusted so it waters the entire round garden. Consequently, flowers that need to be watered more frequently will be placed near the center of the garden, and those that need the least amount of water will be placed farthest from the center. The sectors of the garden will not all be the same size, since they need to accommodate different types of plants.

Here is Madison's design for her garden. The number of degrees in each sector has been marked.



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- Madison has only marked the degree measure on the arcs of the outermost ring of the garden. Determine the angle measure for the arcs on the inner and middle rings of the garden. *They would be the same as the outside arc measure*
- Madison needs to order pavers for the garden. She plans to vary the size and colors of the pavers in different parts of the garden. Consequently, she needs to know the lengths of different portions of the paths. Help her complete this table by calculating the missing arc lengths.

	Distance from Center	Arc Length				
		40° Sector	50° Sector	60° Sector	90° Sector	120° Sector
Inner Circle of Pavers	10 feet	6.9813	8.7266	10.4720	15.7080	20.9440
Middle Circle of Pavers	20 feet	13.9626	17.4533	20.9440	31.4159	41.8879
Outer Circle of Pavers	30 feet	20.9440	26.1799	31.4159	47.1239	62.8319

- As Madison filled out the table she began to notice some interesting things. What did you notice?

*Explain what you notice*

- One thing Madison noticed involved the ratio of the arc length to the radius of the circle. Complete this version of the table and state what you think Madison noticed.

	Distance from Center	Arc length / Radius				
		40° Sector	50° Sector	60° Sector	90° Sector	120° Sector
Inner Circle of Pavers	10 feet	.6981	.8727	1.047	1.5708	2.0944
Middle Circle of Pavers	20 feet	.6981	.8727	1.047	1.5708	2.0944
Outer Circle of Pavers	30 feet	.6981	.8727	1.047	1.5708	2.0944



As Madison examined these numbers, she realized that they behave the same way that degree measurements behave—all arcs in the same sector have the same degree measurement, and all arcs in the same sector have the same value for the ratio of arc length to radius. This made her wonder if these new numbers could be used as a way of measuring angles just as degrees are used.

Later that evening Madison shared her discovery with her older sister Katelyn who is taking calculus at a local university. Katelyn told Madison that her new numbers for measuring angles in terms of the ratio of the arc length to the radius are know as *radians* and that they make the rules of calculus much easier than if angles are measured in degrees.

Madison learned so much from examining the arc length of the sectors of her garden that she decides to examine the areas of the sectors also.

- 5. Complete this table for Madison by calculating the areas of the sectors for the different rings of the garden.

	Distance from Center	Area of Sector				
		40° Sector	50° Sector	60° Sector	90° Sector	120° Sector
Inner Circle of Pavers	10 feet	34.9066	43.6332	52.3599	78.5398	104.7198
Middle Circle of Pavers	20 feet	139.6263	174.5329	209.4395	314.1593	418.8790
Outer Circle of Pavers	30 feet	314.1593	392.6991	471.2389	706.8583	942.4778

- 6. Do you notice anything interesting in this table?

Explain what you notice