Answer Key

7.9 Rays and Radians

A Solidify and Practice Understanding Task

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In the previous task, *Madison's Round Garden*, Madison found a new way to measure angles. Apparently Madison was not the first person to have this idea of measuring an angle in terms of arc length, but once she was aware of it she decided to examine it further.

Here are some of Madison's questions. See if you can answer them.

1. Since a 40° angle measures 0.698 radians (to the nearest thousandth), a 50° angle measures 0.873 radians, and a 60° angle measures 1.047 radians, what angle, measured in degrees, measures 1.000 radian?

$$\frac{\frac{d}{360}\cdot 2\cdot \pi \cdot \cancel{r}}{\cancel{r}} = 1 \rightarrow \frac{d}{180}\cdot \pi = 1 \rightarrow d\pi = 180 \rightarrow d = \frac{180}{\pi} \approx 57.3^\circ$$

2. A circle measures 360°. How many radians is that?

$$\frac{360^\circ}{180^\circ}\cdot \pi = 2\pi \approx 6.28 \text{ radians}$$

3. The formula Madison has been using to calculate radian measurement for an angle that measures $n°$ on a circle of radius r is $\dfrac{\frac{n^\circ}{360^\circ}(2\pi r)}{r} = x \ radians$.

Is there a simpler formula for converting degree measurement to radian measurement?

$$\frac{n^\circ}{360^\circ}\cdot 2\pi = \frac{n^\circ}{180^\circ}\cdot \pi = x \text{ radians}$$

4. What formula might you use to convert radian measurement back to degrees?

$$\frac{n^\circ}{180^\circ}\cdot \pi = x \rightarrow \frac{x\cdot 180^\circ}{\pi} = n^\circ$$

Madison is so excited about radian measurement she decides to learn more about it by going online. At http://en.wikipedia.org/wiki/Radian she finds this statement: *An arc of a circle with the same length as the radius of that circle corresponds to an angle of 1 radian. A full circle corresponds to an angle of 2π radians.*

5. Why is the first sentence in this statement true?

Give explanation

6. Why is the second sentence in this statement true?

Give explanation

Madison finds this idea of writing radian measurement in terms of π appealing. Since a circle measures 2π radians, she reasons that half of a circle, 180°, would measure π radians; and that a quarter of a turn, a right angle, would measure $\pi/4$ radians. Suddenly Madison realizes that while she has been deep in thought thinking about this new idea, she has been fiddling with her protractor. Now her attention focuses on this tool for measuring angles.

Like Madison, you have probably used a protractor to measure angles. A protractor is usually marked to measure angles in degrees. Madison decides she would like to create a protractor to measure angles in radians.

7. Label the following protractor in radians, using fractions involving π. You should label every 10° from 0° to 180°. For example, rays passing through the 0° and 40° angle mark would form an angle measuring $\frac{2}{9}\pi$ (or $\frac{2\pi}{9}$) radians, so we would label the tic mark at 40° as $\frac{2\pi}{9}$.