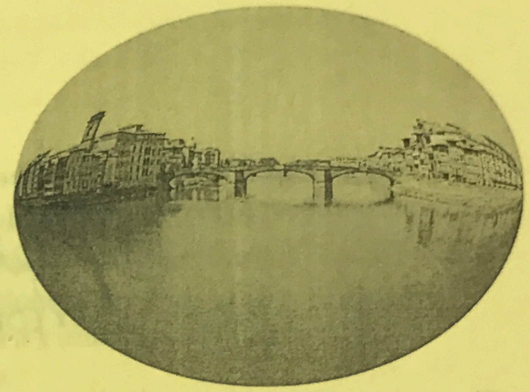
8.7H Operating on a Shoestring

A Solidify Understanding Task



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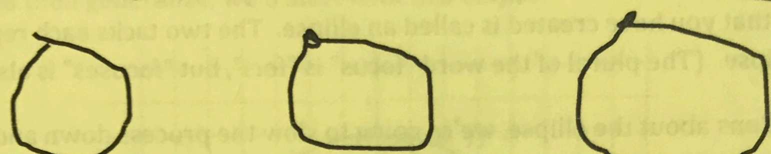
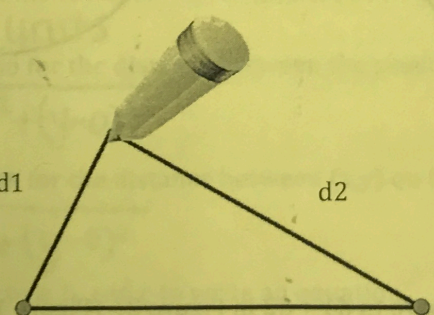
You will need 3 pieces of paper, a piece of cardboard that is at least 8" x 8", 2 tacks, 36 inches of string, and a pencil

1. Cut three pieces of string: a 10 inch piece, a 12 inch piece, and a 14 inch piece. Tie the ends of each piece of string together, making 3 loops.



2. Place a piece of paper on top of the cardboard.
3. Place the two tacks 4 inches apart, wrap the string around the tacks and then press the tacks down.
4. Pull the string tight between the two tacks and hold them down between your finger and thumb. Pull the string tight so that it forms a triangle, as shown below. What is the length of the part of the string that is not on the segment between the two tacks, the sum of the lengths of the segments marked d1 and d2 in the diagram?
5. With your pencil in the loop and the string pulled tight, move your pencil around the path that keeps the string tight.



6. What shape is formed? What geometric features of the figure do you notice?

An oval, rounded closed figure

7. Repeat the process again using the other strings. What is the effect of the length of the string?

Ovals get bigger and more circular

8. What is the effect of changing the distance between the two tacks? (You may have to experiment to find this answer.)

IF the tacks get farther apart, the oval gets more narrow.

The geometric figure that you have created is called an ellipse. The two tacks each represent a focus point for the ellipse. (The plural of the word "focus" is "foci", but "focuses" is also correct.)

To focus our observations about the ellipse, we're going to slow the process down and look at points on the ellipse in particular positions. To help make the labeling easier, we will place the ellipse on the coordinate plane.

9. The distances from the point on the ellipse to each of the two foci is labeled d1 and d2.

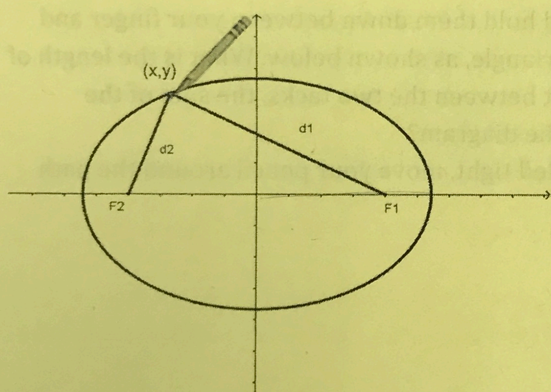


Figure 1

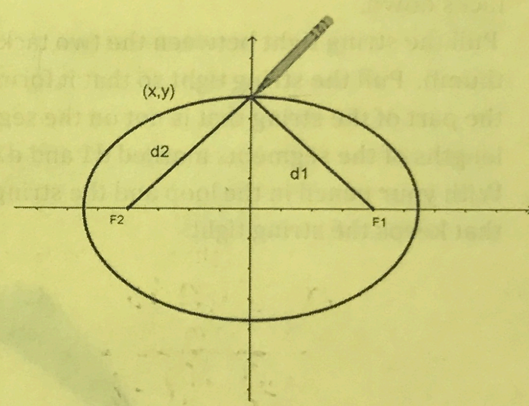


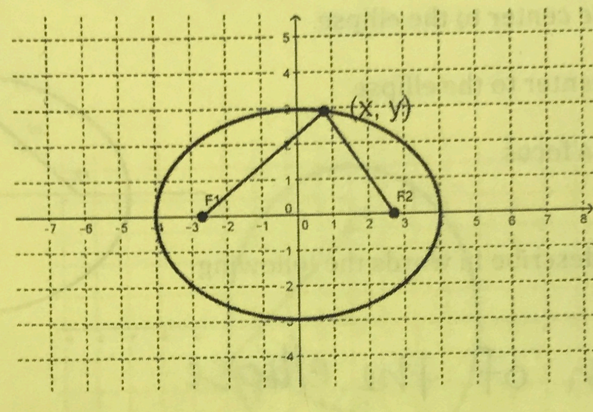
Figure 2

How does d1 + d2 in Figure 1 compare to d1 + d2 in Figure 2? (Figure 1 and Figure 2 are the same ellipse.) They are equal

10. How does d1 + d2 compare to the length of the ellipse, measured from one end to the other along the x-axis? Explain your answer with a diagram.

The distances are equivalent. Look through the ellipses on the next page.

You have just constructed an ellipse based upon the definition: An ellipse is the set of all points *(x, y)* in a plane which have the same total distance from two fixed points called the foci. Like circles and parabolas, ellipses also have equations. The basic equation of the ellipse is derived in a way that is similar to the equation of a parabola or a circle. Since it's usually easier to start with a specific case and then generalize, we'll start with this ellipse:



11. Now, use the conclusions that you drew earlier to help you to write an equation. (We'll help with a few prompts.)

a. What is the sum of the distances from a point (x, y) on this ellipse to F1 and F2?

About 8 units

b. Write an expression for the distance between the point (x, y) on the ellipse and F1(-3,0).

$D=\sqrt{(x+3)^2+(y-0)^2}$

c. Write an expression for the distance between (x,y) on the ellipse and F2 (3,0).

$D=\sqrt{(x-3)^2+(y-0)^2}$

~~d.~~ Use your answers to a, b, and c to write an equation.

12. The equation of this ellipse in standard form is:

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

It might be much trickier than you would imagine to re-arrange your equation to check it, so we'll try a different strategy. This equation would say that the ellipse contains the points (4,0) and (0,-3). Do both of these points make your equation true? Show how you checked them here.

$(0,-3) \rightarrow \frac{(0)^2}{16} + \frac{(-3)^2}{9} = 0 + \frac{9}{9} = 1$

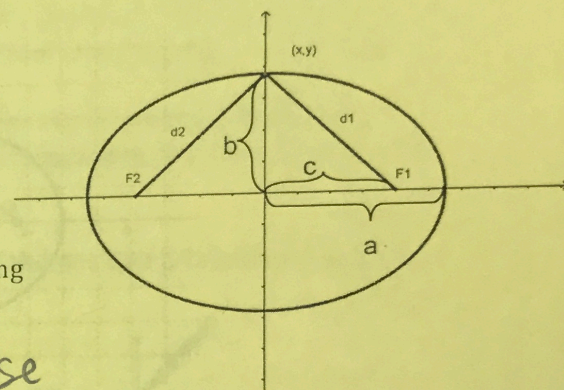
$(4,0) \rightarrow \frac{(4)^2}{16} + \frac{(0)^2}{9} = \frac{16}{16} + 0 = 1$

13. Using the standard form of the equation is actually pretty easy, but you have to notice a few more relationships. Here's another picture with some different parts labeled.

a = horizontal distance from the center to the ellipse

b = vertical distance from the center to the ellipse

c = distance from the center to a focus



Based on the diagram, describe in words the following expressions:

2a the length of the ellipse

2b the height of the ellipse

2c the distance between foci

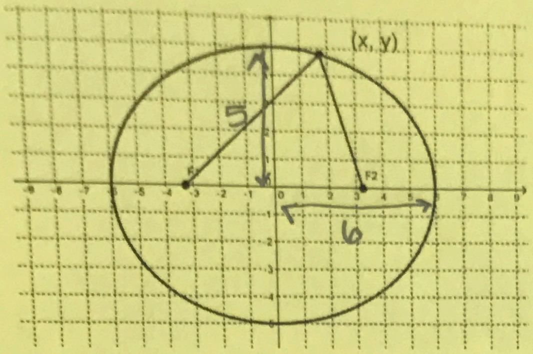
14. What is the mathematical relationship between a, b, and c?

give your own observation

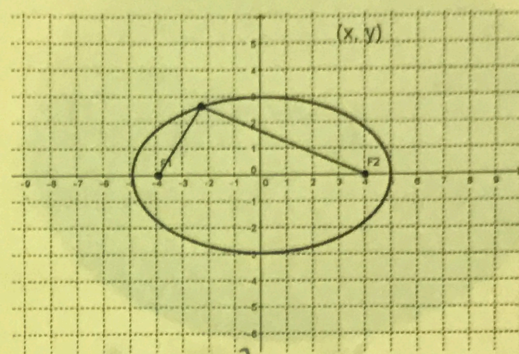
15. Now you can use the standard form of the equation of an ellipse centered at (0,0) which is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

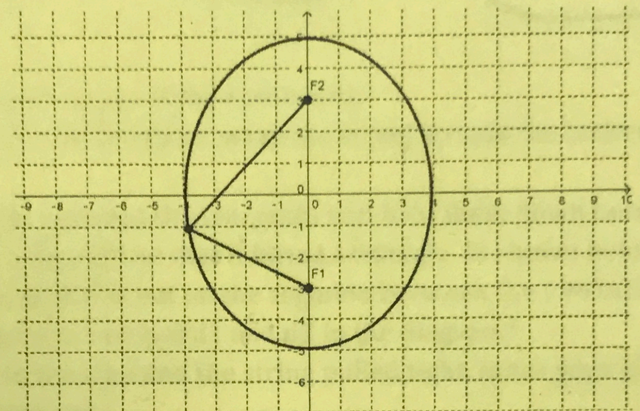
Write the equation of each of the ellipses pictured below:



$$\frac{x^2}{36}+\frac{y^2}{25}=1$$

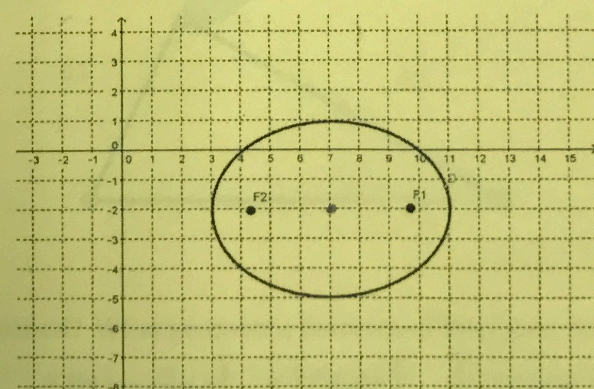


$$\frac{x^2}{25}+\frac{y^2}{9}=1$$



$$\frac{x^2}{16}+\frac{y^2}{25}=1$$

16. Based on your experience with shifting circles and parabolas away from the origin, write an equation for of this ellipse. Test your equation with some points on the ellipse that you can identify.



$$\frac{(x-7)^2}{16}+\frac{(y+2)^2}{9}=1$$