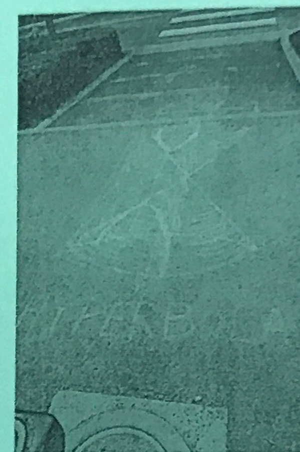
8.8H What Happens If . . .?

A Solidify Understanding Task



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After spending some time working with circles and ellipses, Maya notices that the equations are a lot alike. For example, here's an equation of an ellipse and a circle:

$$\frac{x^2}{16}+\frac{y^2}{9}=1 \qquad x^2+y^2=16$$

1. What are some of the similarities between the circle and the ellipse given in the equations above? What are some of the differences?

List 3 similarities and 3 differences

2. Maya wonders what would happen if she took the equation of the circle and rearranged it so the right hand side was 1, like the standard form of an ellipse. What does the equation of the circle become?

$$\frac{x^2+y^2}{16}=\frac{16}{16} \rightarrow \frac{x^2}{16}+\frac{y^2}{16}=1$$

3. After seeing this equation Maya wonders if a circle is really an ellipse, or if an ellipse is really a circle. How would you answer this question?

A circle is a special ellipse where the two focus points are in the same spot

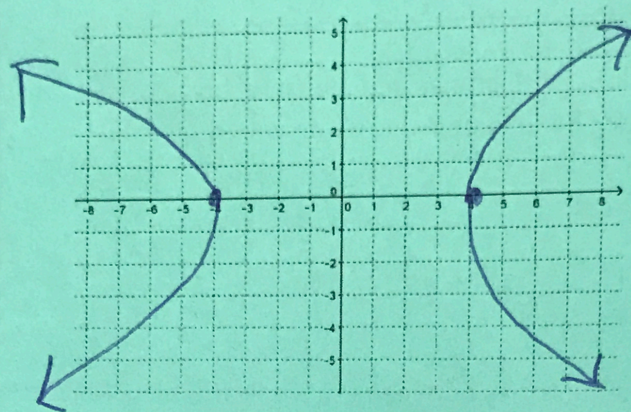
4. Maya looks at the equation of the ellipse and wonders what would happen if the "+" in the equation was replaced with a "—", making the equation:

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

Without making any further calculations or graphing any points, predict whether or not the graph of this equation will be an ellipse? Using what you know about ellipses, explain your answer.

Make a prediction

5. Graph the equation to determine whether or not your prediction was correct. Be sure to use enough points to get a full picture of the figure.



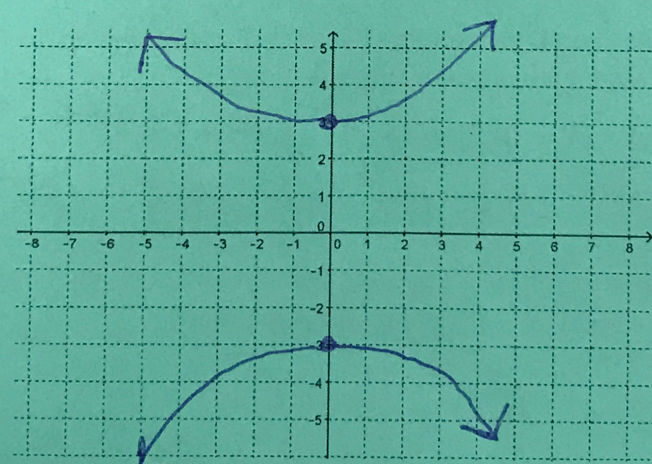
6. What are some of the features of the figure that you have graphed?

List 3 Features you See

7. Maya's teacher tells her that the name of the figure represented in each of the two equations is a hyperbola. Maya wonders what would happen if the x^2 term in the equation was switched with the y^2 term, making the equation:

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

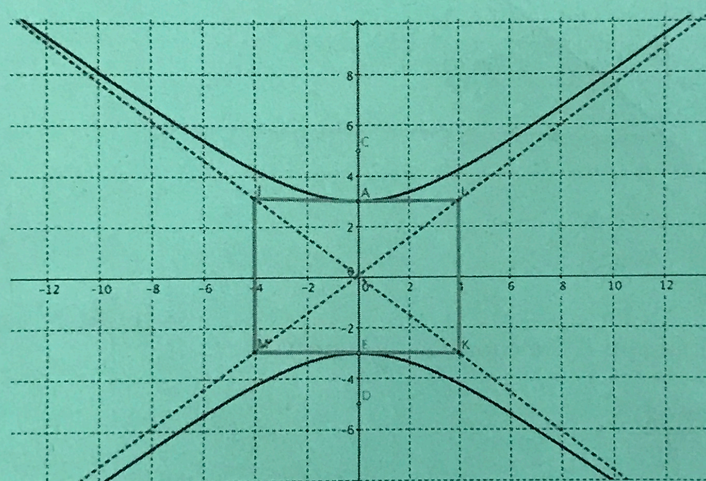
Graph this equation and compare it to the hyperbola that you graphed previously.



8. What similarities and differences do you see between this hyperbola and the one that you graphed in #5?

List 3 similarities and 3 differences

One strategy that makes it easier to graph the hyperbola from an equation is to notice that the square root of the numbers under the x^2 and y^2 terms can be used to make a rectangle and then to draw dotted lines through the diagonals that form the boundaries of the hyperbola. Using this strategy to graph the equation: $\frac{y^2}{9} - \frac{x^2}{16} = 1$, you would start by taking the square root of 9 = 3 and going up and down 3 units from the origin. Then you take the square root of 16 = 4 and go left and right 4 units from the origin. Make a rectangle with these points on the sides and draw the diagonals. You will get this:



9. So, Maya, the bold math adventurer, decides to try it with a new equation of a hyperbola. The standard form of the equation of an hyperbola centered at (0,0) is:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (opens left and right)

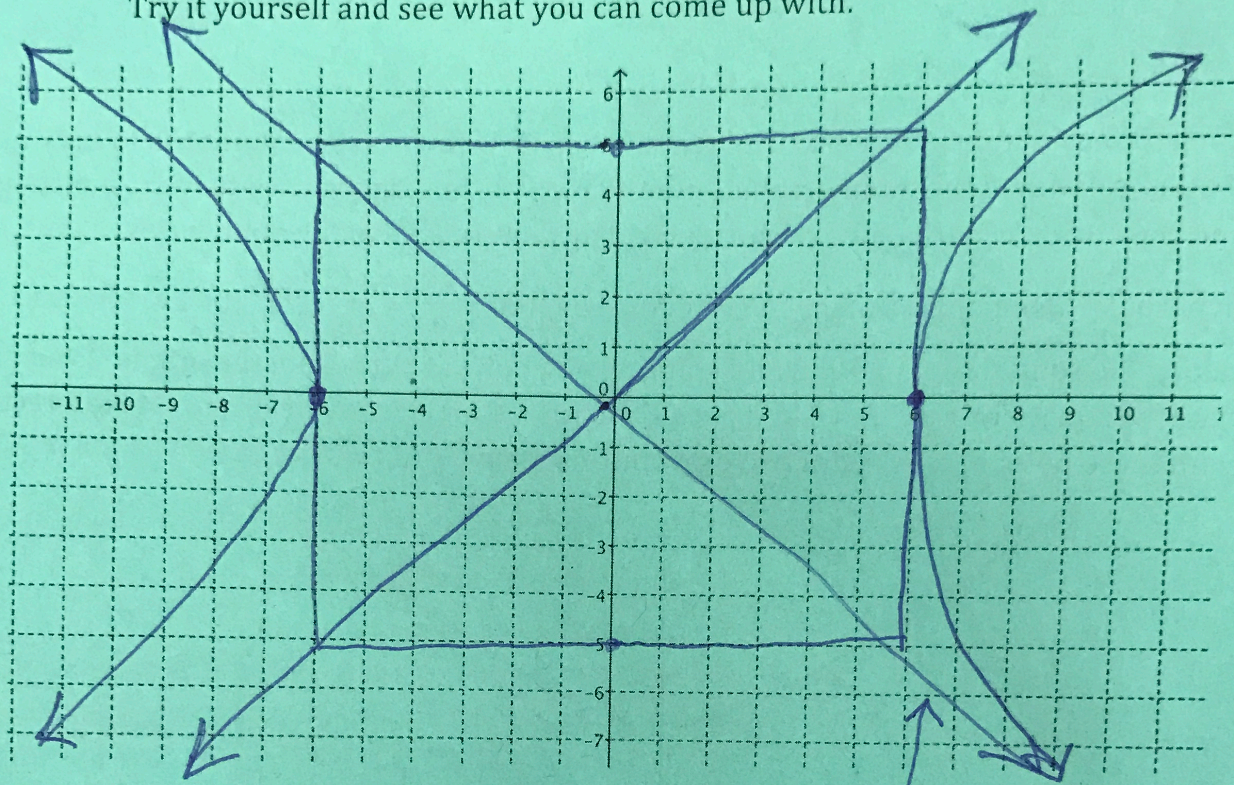
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (opens up and down)

It is determined by which term is negative

Maya goes to work graphing the equation:

$$\frac{x^2}{36} - \frac{y^2}{25} = 1$$

Try it yourself and see what you can come up with.



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10. Maya wonders what happens if the equation becomes:

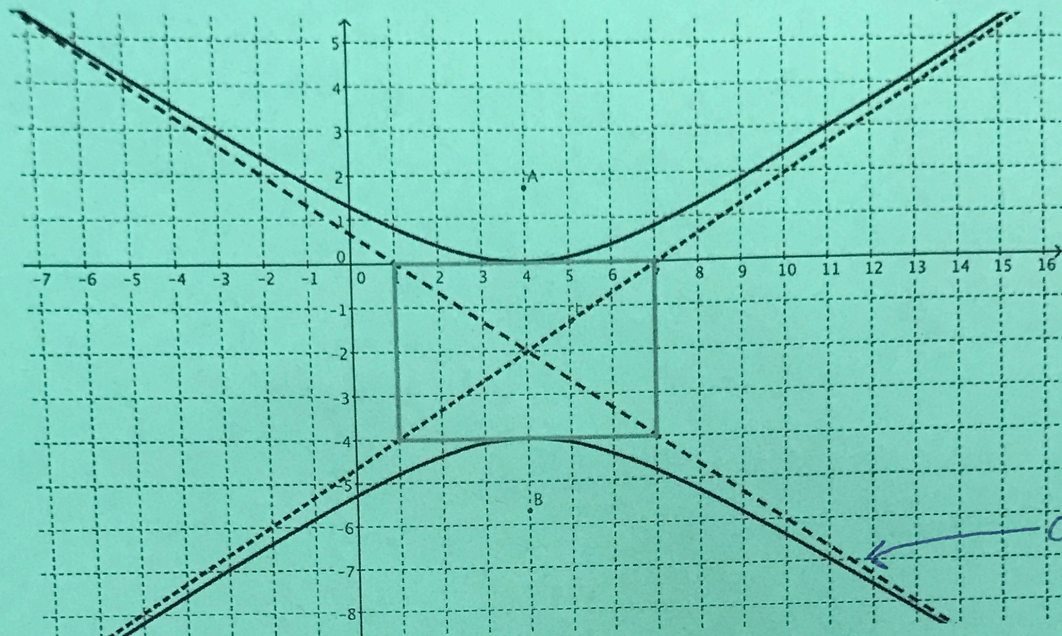
$$\frac{(x-1)^2}{36} - \frac{(y+2)^2}{25} = 1$$

What is your prediction? Why?

Give your prediction and explain

11. Write the equation of the hyperbola shown below: $-\frac{(x-4)^2}{9} + \frac{(y+2)^2}{4} = 1$ or

$\frac{(y+2)^2}{4} - \frac{(x-4)^2}{9} = 1$



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11. What similarities and differences do you see between a hyperbola and an ellipse?

List 3 of each.